Dr. Helmut Quabeck
Finkenweg 39
64832 Babenhausen
Germany

HQ-Modellflugliteratur

# On the Longitudinal Stability of Gliders

By Helmut Quabeck, ©

## *Abbreviations*

($\boldsymbol{e_1, e_2, e_3}$) experimental system of coordinates

$c$ = chord length

$\hat{c}$ = mean aerodynamic chord length (MAC)

$c_l$ = lift-coefficient of the profile

$c_L$ = lift-coefficient

$c_{Lw}$ = lift-coefficient of the wing

$c_{Lh}$ = lift-coefficient of the elevator

$c_d$ = drag-coefficient of the airfoil

$c_{Dw}$ = drag coefficient of the wing

$c_{Di}$ = induced drag-coefficient of the wing

$c_{mo}$ = momentum-coefficient of the airfoil related to its aerodynamic centre

$c_M$ = momentum-coefficient of plane

$c_{Mo}$ = momentum-coefficient of the lifting wing related to its aerodynamic centre

$r_h$ = distance of the aerodynamic centre of the elevator from the c.g.

$\boldsymbol{X,Y,Z}$ = inertia-forces in the experimental-system

$A$ = wing-area

$A_h$ = elevator-area

$L$ = lift

$L_w$ = lift of the wing

$L_h$ = lift of the elevator

$D$ = drag

$D_w$ = drag of the lifting wing

$D_i$ = induced drag

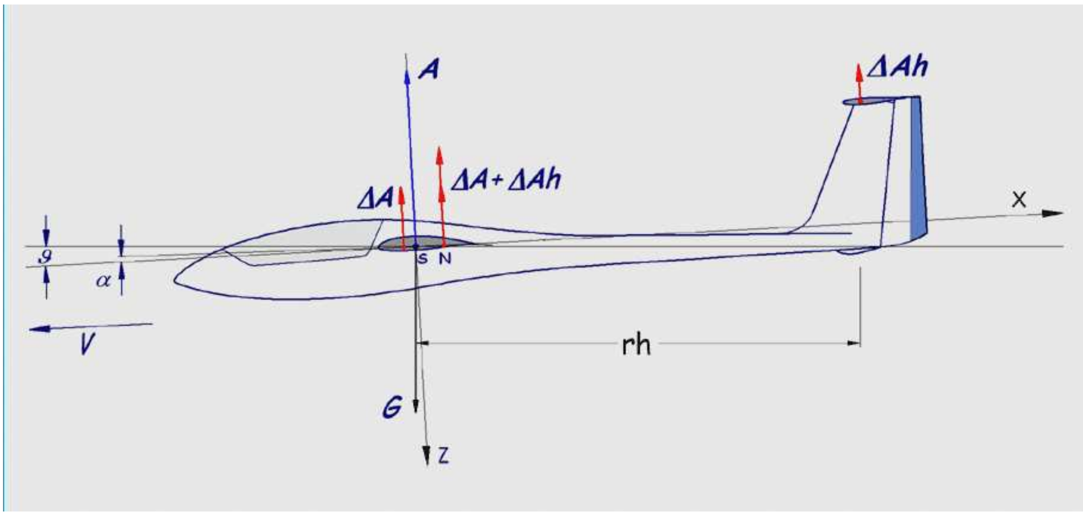
$M$ = pitching-moment of the glider

$Mo$ = pitching-moment of the wing related to the aerodynamic centre

$X_N$ = position of the aerodynamic centre of the glider

$X_{Nw}$ = position of the aerodynamic centre of the lifting wing

| | | |
|---|---|---|
| Xc.g. | = | position of the centre of gravity |
| V | = | velocity of the glider |
| $\alpha$ | = | angle of attack |
| $\alpha_w$ | = | downwash-angle |
| $\varepsilon$ | = | difference of the angles of incidence of elevator and wing |
| $\acute{\alpha}$ | = | $d\alpha/dt$ rotational speed of the angle of attack |
| $\vartheta$ | = | angle of inclination, gliding-angle |
| $\varepsilon$ | = | difference of angles of incidence |
| $\Lambda_w$ | = | aspect-ratio of the wing |
| $a_w$ | = | lift efficiency-factor of the lifting wing |
| $a_p(\alpha)$ | = | airfoil efficiency-factor of the lifting wing |
| $a_w^x$ | = | efficiency-factor of the lifting wing, $a_w^x = a_w \cdot a_p(\alpha)$ |
| $\Lambda_h$ | = | aspect-ratio of the elevator (shape influence) |
| $a_h$ | = | lift efficiency-factor of the elevator (shape-influence) |
| q | = | $\omega_y$ = rotational speed around the lateral y-axis through the c.g. |
| $\bar{q}$ | = | aerodynamic pressure, $\bar{q} = \rho/2 \cdot V^2$ |
| $\rho$ | = | airdensity, $\approx 1.25$ kg/m$^3$ at sea-level |
| $\sigma$ | = | measure of the static stability |
| m | = | body-mass |
| $m_i$ | = | mass of model part i (e.g. wing, fuselage, elevator,…) |
| $r_i$ | = | distance of the mass-centre of model-part i from the c.g. |
| $J_y$ | = | mass-moment of inertia related to y-axis (c.g.) |
| $J_{yi}$ | = | mass-moment of inertia of model-part i related to y-axis (c.g.) |
| F(n) | = | characteristic equation with n solutions |
| $\lambda_{1;2}$ | = | solutions of F(4) for $\alpha$-disturbances- |
| $\lambda_{3;4}$ | = | solutions of F(4) for $\vartheta$-V-disturbances |




---

## 1. General Determinations

Generally in this paper for all theoretical considerations the so-called experimental system (***e₁***, ***e₂***, ***e₃***) of coordinates will be chosen , where $\boldsymbol{e_1}$ coincides with the gliding path of the model and has an angle $\vartheta$ of inclination against the geodetic horizon. $\boldsymbol{e_2}$ is chosen horizontally and coincides with the lateral-y-axis of the glider in wing direction, and $\boldsymbol{e_3}$ is defined by $\boldsymbol{e_3} = \boldsymbol{e_1} \times \boldsymbol{e_2}$.

The angle of attack $\alpha$ of the wing thus is related to the axis $\boldsymbol{e_1}$.

The angles $\alpha$ and $\vartheta$ generally can vary independent on each other and consequently also their time derivatives $\dot{\alpha} = d\alpha/dt$ and $q = \omega_y = d\vartheta/dt$.

Consequently the direction of the gliding speed V points towards the negative direction of the $\boldsymbol{e_1}$ –axis.

## 2. Forces at the Glider at longitudinal Motion

In the experimental system of coordinates the equations of the forces acting at a glider, namely the forces due to mass inertia and the aerodynamic lift and drag forces, are

$\boldsymbol{e_1}$ – direction:

$$X = m \cdot \dot{V}$$

$$X = -m \cdot g \cdot \sin\vartheta - D \qquad (2.1)$$

– $\boldsymbol{e_3}$ – direction:

$$Z = m \cdot V \cdot \dot{\vartheta}$$

$$Z = -m \cdot g \cdot \cos\vartheta + L \qquad (2.2)$$

## 3. Pitching-Moment of the Glider

The pitching-moment M caused by air forces at the glider in general depends on

- the angle of attack $\alpha$,
- the rotational speed of the angle of attack $\dot{\alpha} = d\alpha/dt$ ,
- and the rotational speed around the y-axis (lateral axes) $q = \omega_y$

$$\boxed{M = M(\alpha, \dot{\alpha}, \omega_y)} \qquad (3.1)$$

Using a non-dimensional momentum coefficient $c_m$, according to aerodynamic theory we get

$$\boxed{M = c_m(\alpha, \dot{\alpha}, \omega_y) \cdot \bar{q} \cdot A \cdot \hat{c}} \qquad (3.2)$$

Taylor development of the momentum-coefficient $c_m$ provides

$$\boxed{c_m = c_m(\alpha, 0, 0) + \frac{\dot{\alpha} \cdot \hat{c}}{V} \cdot c_{m,\dot{\alpha}} + \frac{\omega_y \cdot \hat{c}}{V} \cdot c_{m,\omega_y}} \qquad (3.3)$$

Therein the derivatives $c_{m\dot{\alpha}} = \partial c_m / \partial \dot{\alpha}$ and $c_{m\omega y} = \partial c_m / \partial q$ are dependent on $\alpha$.

The second term signifies the coefficient of a damping-moment which is proportional to the rotational speed $\dot{\alpha}$ . Essentially it results from the fact that the angle $\alpha_w(t)$ of the downwash resulting from the lifting wing at the elevator in case of $\dot{\alpha} \neq 0$ does not correspond to the angle of attack $\alpha(t)$ of the wing, but to the angle $\alpha(t + \Delta t)$. $\Delta t \approx r_h/V$ and $r_h$ is about the distance of the aerodynamic centre of the elevator from the centre of gravity, c.g. E.g. for $\dot{\alpha} > 0$ the downwash angle $\alpha_w$ gets smaller and the resulting angle of attack at the elevator becomes $\alpha_h = \alpha(t) - \alpha_w(t + \Delta t)$. Thus the lift at the elevator becomes larger then for the stationary case with $\dot{\alpha} = 0$, a negative pitching-moment will result, and in general we have $c_{m\dot{\alpha}} < 0$.

The third term signifies the coefficient of a dampening-moment which is proportional to the rotational speed $q = \omega_y$ around the lateral axis through the c.g. It results from the fact that the angle of attack and thus the lift at the elevator in case of $\omega_y > 0$ is increased by the angle $\omega_y \cdot \hat{c}/V$ as compared to the stationary state with $\omega_y = 0$. Again a negative pitching-moment results with $c_{m\omega y} < 0$.

These aerodynamic pitching-moments are counteracted by the mass-inertia of the glider-parts expressed by their moments of inertia $J_y$ around the y-axis of the glider**.** May $m_i$ be the mass of a glider part and $r_i$ its distance from the c.g., then the mass-moment of inertia of the glider can roughly be assessed to be

$$J_y = \sum_i m_i \cdot r_i^2 \tag{3.4}$$

Thus for the pitching moment generally we have

$$M = J_y \cdot \left(\frac{\partial^2 \vartheta}{\partial t^2} + \frac{\partial^2 \alpha}{\partial t^2}\right) = J_y \cdot \left(\frac{\partial q}{\partial t} + \frac{\partial \dot{\alpha}}{\partial t}\right) \tag{3.5}$$

## 4. Aerodynamic Centre of the Glider, $X_N$

The aerodynamic centre of the glider is defined as the point $X_N$ on its longitudinal axis about which the pitching moment is constant with respect to the angle of attack. Thus in case of a change of the angle of attack the resulting Lift L will act through $X_N$.

Equally the aerodynamic centre of the lifting wing is defined as the point $X_{Nw}$ about which the pitching moment of the wing, in generally dependent on the wing-shape and the properties of the chosen airfoils, is constant with respect to the angle of attack. Since the lift- and momentum-characteristics, $c_l(\alpha)$ and $c_m(\alpha)$, of most airfoils show a slightly non-linear dependence on the angle of attack for low Reynold-numbers, $Re < 1\cdot10^6$, the aerodynamic centre of model-wings can only be considered to be stable for small $\alpha$-ranges. This has to be taken into account at the design of a model-plane in order to achieve proper static and dynamic flight behaviour under all flight conditions.

The displacement of the overall aerodynamic centre of the plane $X_N$ vs. the aerodynamic centre of the wing $X_{Nw}$ may be denoted by $\Delta X_N = X_N - X_{Nw}$. The momentum-equilibrium of the plane is then given by

$$\Delta X_N \cdot L = r_{Nh} \cdot L_h \tag{4.1}$$

Herein $L_h$ is the contribution of the elevator to the overall lift of the plane, and $r_{Nh}$ is the distance of the aerodynamic centres of lifting wing and elevator. According to aerodynamic theories we get

$$L_h = c_{lh} \cdot A_h \cdot q_h \tag{4.2}$$

$c_{lh}$ is the lift coefficient of the elevator related to the area $A_h$ of the elevator and the aerodynamic pressure $q_h$ at the location of the elevator. For practical reasons it is preferred to relate the lift coefficient of the elevator to the wing area A and its aerodynamic pressure $q$ :

$$L_h = c_{Lh} \cdot A \cdot q \tag{4.3}$$

By comparison one gets

$$c_{Lh} = c_{lh} \cdot A_h / A \cdot q_h / q \qquad (4.4)$$

If the airflow on the elevator would not be influenced by the wake from the wing we would get the derivative

$$c_{Lh,\alpha} = \frac{\partial c_{Lh}}{\partial \alpha} \cdot \alpha = \frac{\partial c_{lh}}{\partial \alpha} \cdot \alpha \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.5)$$

However, since downwash **w** is generated in the wake of the wing, the angle of attack at the elevator location is reduced by the downwash angle $\alpha_w = w/V$. The difference of the angles of incidence between wing and elevator may be denoted by $\varepsilon$ , then the angle of attack at the elevator is given by

$$\alpha_h = \alpha + \alpha_w + \varepsilon \qquad (4.6)$$

Taking this angle of attack into account, the coefficient of the elevator lift results to be

$$c_{Lh} = c_{lh,\alpha} \cdot (\alpha + \alpha_w + \varepsilon) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.7)$$

At a disturbance of the longitudinal motion of the plane around the pitching-axis with fixed rudder the change of the elevator lift coefficient with $\alpha$ is given by the derivative $c_{Lh,\alpha}$:

$$c_{Lh,\alpha} = c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.8)$$

The overall lift-coefficient is

$$c_L = c_{Lw} + c_{Lh} \qquad (4.9)$$

and in case of a disturbance around the pitching axis the lift-slope $c_{L,\alpha}$ of the whole plane in case of fixed controls results to

$$c_{L,\alpha} = c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.10)$$

Using formulas 4.8 and 4.10 in formula 4.1, we will receive

$$\Delta X_N = \frac{c_{Lh,\alpha}}{c_{L,\alpha}} \cdot r_{Nh} \qquad (4.11)$$

Related to the mean aerodynamic chord of the wing finally results

$$\frac{\Delta X_N}{\hat{c}} = \frac{c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}}{c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}} \cdot \frac{r_{Nh}}{\hat{c}} \qquad (4.12)$$

In a larger distance behind the wing the free vortices of both halves of the wing induce a downwash angle of about $\alpha_{w\infty} = -2\, c_{Lw} / (\pi \cdot \Lambda_w)$. There from results $\partial \alpha_{w\infty} / \partial \alpha = -2\, c_{Lw,\alpha} / (\pi \cdot \Lambda_w)$ and for the aerodynamic centre of the plane

$$\frac{\Delta X_N}{\hat{c}} = \frac{c_{lh,\alpha} \cdot (1 - \frac{2c_{Lw,\alpha}}{\pi \cdot \Lambda_w}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}}{c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 - \frac{2c_{Lw,\alpha}}{\pi \cdot \Lambda_w}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}} \cdot \frac{r_{Nh}}{\hat{c}} \qquad (4.13)$$

This formula is still rather complex and for most modellers impossible to solve. A way out of this dilemma is found for practical cases when considering how the derivatives $c_{Lw,\alpha}$ and $c_{lh,\alpha}$ depend on the lifting characteristics of the chosen airfoils and on the shape of wing and elevator.

The slope of the lift-coefficient of a lifting wing, namely $c_{Lw,\alpha}$, is closely related to the slope of the lift-coefficient of the applied airfoil, namely $c_{l,\alpha}$ by an efficiency factor denoted as $a_w$

$$c_{Lw,\alpha} = a_w \cdot c_{l,\alpha} \qquad (4.14)$$

$a_w$ takes into account the influence of the wing-shape on the formation of the free vortices on the wing-surfaces. According to the limited wing-span and the wing-shape the ideal lifting-efficiency of the airfoil is reduced. In an ideal non-viscous environment the slope of an ideal airfoil would be $c_{l,\alpha} = 2 \cdot \pi$. However, in a viscous airflow for Re-numbers below $1 \cdot 10^6$ non-linear deviations from this ideal slope may be experienced, in the practically non-critical range of the angles of attack mostly an increase up to 5% are experienced. This can be taken into account by another efficiency-factor $a_p(\alpha)$, Thus we get

$$c_{Lw,\alpha} = a_w \cdot a_{pw}(\alpha) \cdot 2 \cdot \pi \qquad (4.15)$$

$$c_{lh,\alpha} = a_h \cdot a_{ph}(\alpha) \cdot 2 \cdot \pi \qquad (4.16)$$

Denoting $a^x = a_w \cdot a_p(\alpha)$ formula 2.13 after a few rearrangements can be rewritten to a practically easier form:

$$\frac{\Delta X_N}{\hat{c}} = \frac{a_w^\times \cdot a_h^\times \cdot A_h / A}{1 + a_w^\times \cdot a_h^\times \cdot A_h / A} \cdot \frac{r_{Nh}}{\hat{c}} \qquad (4.17)$$

therein $a_w$ and $a_h$ denote the total lifting-efficiencies of wing and elevator.

***Remark***: *The lifting efficiency factors $a_w$ and $a_h$ can easily be determined by means of the* ***"FMFM"****-program of the author.*

For many practical cases it is sufficient to approximately chose $a_p \approx 1$, and if the aspect-ratio $\Lambda \geq 5$ then according to the expanded lifting line theory the efficiency-factors can be approximated by

$$a^\times = \Lambda / (2 + \sqrt{\Lambda^2 + 4}) \qquad (4.18)$$

According to long time experience, using this approach very reliable result of flight-stability could be practically achieved, as will be explained by an example later on.

## 5. Static longitudinal stability, σ

One of the most important flight mechanical characteristics of a glider is the capability, to redress the balance of the original stationary longitudinal flight state after disturbance of the angle of attack by $\Delta\alpha$ without using controls. A disturbance of the angle of attack causes an increase or decrease of the lift by $\Delta L$ of the plane, if thereby a momentum $\Delta M$ is caused that forces the plane to rotate back to the original state, the plane is featured statically stable. Thus a glider behaves statically stable if generally holds true that

$$dM = -\sigma \cdot dL \qquad (5.1)$$

$\sigma$ is a non-dimensional positive constant factor which is considered as a stability-measure for the stationary longitudinal flight of a glider. The larger it is, the larger the back-leading momentum will be. For $\sigma = 0$ the stability-behaviour of the plane will be indifferent and it will no more be controllable, for $\sigma < 0$ the longitudinal flight of the plane will become instable.

***Note*****:** *As will be shown later, besides $\sigma$ also the mass-moments of inertia Jy of the glider parts are to be taken into account to completely determine the time-dependent motion of a glider back to flight-balance after disturbance*.

Using non-dimensional aerodynamic coefficients we get

$$\sigma = -dc_M / dc_L \qquad (5.2)$$

Based on the explanations in chapter 2 the pitching moment around the c.g. is given by

$$M = -(X_N - X_{c.g.}) \cdot L + M_{ow} - r_h \cdot A_h + M_{oh} \qquad (5.3)$$

Therein $M_{ow}$ is the pitching-moment for the wing at the aerodynamic centre, $M_{oh}$ is that of the elevator, and $r_h$ is the distance of the aerodynamic centre of the elevator from c.g. Moments according to the vertical position of the forces can mostly be neglected for gliders. For the change of the pitching-moment around the c.g. by change of the lift here from results

$$dc_M / dc_L = -(X_N - X_{c.g.}) / \hat{c} \qquad (5.4)$$

Implementing (3.4) in (3.2) yields

$$\boxed{\sigma = (X_N - X_{c.g.}) / \hat{c}} \qquad (5.5)$$

Therewith we have a very useful, quantitative measure for the static stability of the glider, namely the distance of the c.g. from the aerodynamic centre of the plane related to the mean aerodynamic chord of the lifting-wing. Because of the requirement $\sigma > 0$, the c.g. must be positioned in front of $X_N$ in order to achieve longitudinal static flight stability.

As will be discussed in more detail later on, usually the position of the centre of gravity is to be chosen such that the lift coefficient $c_L$ at slow stationary gliding is either adapted to the optimum gliding or to the minimum sinkrate. Once the c.g. is determined by evaluation of the profile- and wing-characteristics, by means of the theoretical considerations in chapter 4 the geometrical parameters of the glider can be chosen such that the required size of $\sigma$ will be achieved. One problem here may be how it can be found out what the appropriate size of $\sigma$ is. The most adequate way is to determine the static stability of one or more representative models which are considered to have good stability-behaviour.

## 6. Free Oscillations of a Glider with Fixed Controls

The static stability-measure $\sigma$ in principle just provides an answer to the question whether or not a glider will behave stable on a stationary linear flight path. However, it does not inform how fast the glider will redress the original stationary balance after any disturbance. In case of static stability we can expect that the glider performs attenuated oscillations. In the most general case a glider may conduct combined $\alpha$- and $\vartheta$-oscillations as well as oscillations of the c.g along the gliding path. As will be outlined later on, in most practical cases the $\alpha$-oscillations are much faster than the c.g.-oscillations and by means of appropriate choice of the glider-design-parameters it can be achieved, that these oscillations are damped to such a degree that the glider returns to balance in a very short time. c.g.-oscillations take longer and cannot so well be damped, however, in practice they can easily be balanced out by proper RC-controlling of the pilot.

In order to determine the behaviour of a glider after disturbance of the angle of attack and/or the gliding angle this movements may be considered as small disturbances $\Delta V$, $\Delta\alpha$ und $\Delta\vartheta$ of a stationary linear flight path. Then the equations for the forces at the glider, given in chapter 2, may be developed into *Taylor*-progressions whereby higher power elements are neglected:

$$\boxed{m \cdot \dot{V} = X_V \cdot \Delta V + X_\vartheta \cdot \Delta\vartheta + X_\alpha \cdot \Delta\alpha}$$

$$\begin{aligned} X_V &= -\partial D / \partial V \\ X_\alpha &= -\partial D / \partial \alpha \\ X_\vartheta &= -m \cdot g \cdot \cos\vartheta \end{aligned} \qquad (6.1)$$

and

$$\boxed{m \cdot V \cdot \dot{\vartheta} = Z_V \cdot \Delta V + Z_\vartheta \cdot \Delta\vartheta + Z_\alpha \cdot \Delta\alpha}$$

$$\begin{aligned} Z_V &= \partial L / \partial V \\ Z_\vartheta &= m \cdot g \cdot \sin\vartheta \\ Z_\alpha &= \partial L / \partial \alpha \end{aligned} \qquad (6.2)$$

Equally the momentum equation of chapter 3 is developed to

$$\boxed{J_y \cdot \left(\frac{d^2\Delta\vartheta}{dt^2} + \frac{d^2\Delta\alpha}{dt^2}\right) = M_V \cdot \Delta V + M_\alpha \cdot \Delta\alpha + M_{\dot{\vartheta}} \cdot \dot{\vartheta} + \overline{M}_{\dot{\alpha}} \cdot \dot{\alpha}}$$

$$\begin{aligned} M_V &= \partial M(\alpha,0,0) / \partial V \cdot q \cdot A \cdot \bar{c} \\ M_\alpha &= \partial M(\alpha,0,0) / \partial \alpha \cdot q \cdot A \cdot \bar{c} \\ M_{\dot{\vartheta}} &= \bar{c} / V \cdot c_{m,\omega} \cdot q \cdot A \cdot \bar{c} \\ \overline{M}_{\dot{\alpha}} &= \bar{c} / V \cdot (c_{m,\dot{\alpha}} + c_{m,\omega}) \cdot q \cdot A \cdot \bar{c} \end{aligned} \qquad (6.3)$$

Rearrangement of the force equations 6.1 and 6.2 and of the momentum equation 6.3 provides

$$\boxed{\begin{array}{llll} (m \cdot \frac{d}{dt} - X_V)\Delta V & -X_\vartheta \cdot \Delta\vartheta & -X_\alpha \cdot \Delta\alpha & = 0 \\ -Z_V \cdot \Delta V & +(m \cdot V \cdot \frac{d}{dt} - Z_\vartheta)\Delta\vartheta & -Z_\alpha \cdot \Delta\alpha & = 0 \\ -M_V \cdot \Delta V & +(J_y \cdot \frac{d^2}{dt^2} - M_{\dot{\vartheta}})\Delta\vartheta & +(J_y \cdot \frac{d^2}{dt^2} - \overline{M}_{\dot{\alpha}} \cdot \frac{d}{dt} - M_\alpha)\Delta\alpha & = \mathbf{0} \end{array}} \qquad (6.4)$$

By means of an exponential description of the disturbances according to

$$\Delta V = \Delta V_o \cdot e^{\lambda t}, \quad \Delta \boldsymbol{\alpha} = \Delta \boldsymbol{\alpha}_o \cdot e^{\lambda t}, \quad \Delta \boldsymbol{\vartheta} = \Delta \boldsymbol{\vartheta}_o \cdot e^{\lambda t}$$

the characteristic equation of the system $\boldsymbol{F_4(\lambda)}$ becomes:

$$\boxed{m^2 \cdot V \cdot J_y \cdot F_4(\lambda) \equiv \begin{vmatrix} m \cdot \lambda - X_V & -X_\vartheta & -X_\alpha \\ -Z_V & m \cdot V\lambda - Z_\vartheta & -Z_\alpha \\ -M_V & Jy \cdot \lambda^2 - M_{\dot{\vartheta}} \cdot \lambda & J_y \cdot \lambda^2 - \overline{M}_{\dot{\alpha}} \cdot \lambda - M_\alpha \end{vmatrix}} \qquad (6.5)$$

In flight-mechanical theories this equation is usually written in subsequent form

$$\boxed{\lambda^4 + B \cdot \lambda^3 + C \cdot \lambda^2 + D \cdot \lambda + E = 0} \qquad (6.6)$$

According to the stability criteria of *Hurwitz* for an oscillating system like the one considered

$$E > 0! \qquad (6.7)$$

is a necessary requirement for the longitudinal stability of the glider. Taking equations 6.1 to 6.3 into account, in detail we get

$$\boxed{m^2 V J_y \cdot E = \left(\frac{\partial D}{\partial V} mg \sin\vartheta - \frac{\partial L}{\partial V} mg\cos\vartheta\right) M_\alpha + \left(\frac{\partial L}{\partial \alpha} mg\cos\vartheta - \frac{\partial D}{\partial \alpha} mg \sin\vartheta\right) M_V} \qquad (6.8)$$

This can be rewritten to

$$m^2 V J_y \cdot E = -\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg \sin\vartheta) \cdot \left[ M_\alpha + M_V \frac{\frac{\partial}{\partial \alpha}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)}{\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)} \right]$$

Here from results

$$\boxed{E = \frac{-\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)}{m^2 V J_y} \cdot \left[ M_\alpha + M_V \cdot \frac{\delta V}{\delta \alpha} \right]} \qquad (6.9)$$

Under normal angles of attack $\partial L/\partial V \cdot m \cdot g \cdot \cos\vartheta - \partial D/\partial V \cdot m \cdot g \cdot \sin\vartheta > 0$, thus the requirement 6.7 is identical with the requirement

$$\boxed{-\left[ M_\alpha + M_V \frac{\delta V}{\delta \alpha} \right] \geq 0} \qquad (6.10)$$

$\delta V/\delta\alpha$ denotes the deviation of the speed by $\alpha$, therefore the differential-quotient of the overall pitching-moment M derived by $\alpha$ and taken along the speed polar must be negative in order to achieve static longitudinal stability:

$$\boxed{-\delta M/\delta\alpha > 0} \qquad (6.11)$$

This result is well in correspondence with those of chapter 5.

## 6.1 Fast pitching-oscillations

At stationary free flight gliding under condition 6.11 for static longitudinal stability, because of the requirements $\Delta V = \Delta\vartheta = 0$ the characteristic equation 6.5 will be reduced to

$$\boxed{J_y \cdot \lambda^2 - \overline{M}_{\dot\alpha} \cdot \lambda - M_\alpha = 0} \qquad (6.1.1)$$

This is the characteristic equation of an attenuated pitch-oscillation around the c.g. Since the attenuation factor $-M_{\dot\alpha}$ is always positive by nature, this equation provides real roots in case of stability with $-M_\alpha > 0$. By means of equations 6.3 we get

$$J_y \cdot \lambda^2 - \frac{\bar{c}}{V}(c_{m,\alpha} + c_{m,\omega y}) \cdot q \cdot A \cdot \hat{c} \cdot \lambda - c_{m,\alpha} \cdot q \cdot A \cdot \hat{c} = 0 \qquad (6.1.2)$$

In order to solve this equation, next the derivatives herein have to be determined.

### 6.1.a The q-derivative

Dependent on the rotation of the glider around the lateral axis y with the angular speed $q = \omega_y$, the so called q-derivatives, will play a roll. They result from the distinct air wash which emerges at the various parts of the glider by interference of the general airflow with speed V and of the local vertical air flow with speed $q \cdot r = \omega_y \cdot r$ of the rotation, and where r is the distance of the glider-part from the c.g. The change of the flow-direction thereupon then corresponds to an incremental angle of attack, also called "dynamic" angle of attack $\alpha_{dyn}$, given by

$$\alpha_{dyn} = \text{atan}\,(q \cdot r / V) \approx q \cdot r / V \qquad (6.1.3)$$

Thereby at the elevator an incremental lift results which is given by

$$\Delta L_h = (c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h}{V} \cdot q_h \cdot A_h \qquad (6.1.4)$$

Therein $r_h$ is the distance of the aerodynamic centre of the elevator from c.g., for the incremental lift coefficient follows

$$\Delta c_L = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot (c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h}{V} \qquad (6.1.5)$$

Taking into account that

$$c_{L,\omega y} = \partial c_L / \partial(\omega_y \cdot \hat{c} / V) \qquad (6.1.6)$$

the q-derivative of the elevator becomes

$$(c_{l,\omega y})_h = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \cdot (c_{l,\alpha})_h \qquad (6.1.7)$$

and for the overall derivative will result

$$c_{L,\omega y} = (c_{L,\omega y})_{wing+fuselage} + \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \cdot (c_{l,\alpha})_h \qquad (6.1.8)$$

The elevator contribution of the pitch attenuation moments $c_{m,\omega y}$ thus is

$$\begin{aligned} \Delta M_h &= -r_h \cdot \Delta L_h \\ &= -(c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h^2}{V} \cdot q_h \cdot A_h \end{aligned} \qquad (6.1.9)$$

With $\Delta M_h = (c_{m,\omega y})_h \cdot \omega_y \cdot (\hat{c}/V) \cdot q \cdot A \cdot \hat{c}$ we get

$$(c_{m,\omega y})_h = -\frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (c_{l,\alpha})_h \qquad (6.1.10)$$

For the overall pitch-attenuation-moment it follows

$$c_{m,\omega y} = (c_{m,\omega y})_{wing+fuselage} - (c_{m,\omega y})_h \qquad (6.1.11)$$

At conventional glider-configurations, low sweep of the lifting wing, and proper elevator distance from c.g., usually the contributions of wing and fuselage are less than 1/10 of the elevator contribution. For most practical cases in model flying we can assume that the aerodynamic pressure at wing and elevator are about equal, $q_h/q \approx 1$, and thus we finally get:

$$c_{m,\omega y} \approx -\frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (c_{l,\alpha})_h \qquad (6.1.12)$$

Using formula 4.16, this attenuation derivative can finally be written in the form

$$c_{m,\omega y} = -2\pi \cdot a_h \cdot a_{ph}(\alpha) \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \qquad (6.1.13)$$

This equation is of major importance for the design of a glider. The efficiency factor $a_h$ pays regard to the geometric shape and to the aspect ratio $\Lambda_h$ of the elevator, according to the extended lifting-line-theory with good approximation $a_h \approx \Lambda_h/(2+(\Lambda_h^2+4)^{1/2})$, e.g. for an elevator with $\Lambda_h = 6$ we roughly get $a_h \approx 0.72$. As described in chapter 4 the factor $a_{ph}$ pays regard to the viscous flow-effects at the elevator-airfoil on its slope of the lift-coefficient with $\alpha$. Usually the c.g of a glider is chosen for optimum gliding or minimum sinkrate, then the lift at the elevator is close to zero and accordingly also the angle of attack. In order to increase the velocity of the glider, an increase of the angle of attack is required at the elevator. Roughly, most commonly used airfoils of elevators have symmetrical shape and their viscosity-factor at lower Re-numbers may deviate considerably from the ideal value $a_{ph} \approx 1$ for high speed. Thus, in order to guarantee a distinct pitch-attenuation-derivative $c_{m,\omega y}$ at any possible flight-velocity, for $a_{ph}$ the minimum $a_{ph} \approx 1$ should be chosen and the parameters $a_h$, $A_h$ and $r_h^2$ of the elevator accordingly be adapted. This means, for most practical purposes it is sufficient to use the equation

$$c_{m,\omega y} = -2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \qquad (6.1.13a)$$

In many cases the value of $c_{m,\omega y}$ can be adopted from models known to provide good attenuation behaviour.

## 6.1.b The ά - derivative of $c_m$

The attenuation-derivative by $\acute{\alpha} = d\alpha/dt$ on the one side takes the retarded new formation of the airflow from the lifting wing into account which results from an accelerated movement of the angle of attack, on the other side it pays regard to the downwash-fraction arriving at the elevator with delay after an non-stationary airflow-changes at the wing. With change of the angle of attack by $\Delta\alpha$ in the first instance there will appear equal $\alpha$-changes at wing and elevator. But only after a time delay of $\Delta t$ the downwash-change of the lifting wing becomes effective at the elevator what then leads to an incremental change of the angle of attack of the elevator. Under stationary flight-conditions $(\partial\alpha_w/\partial\alpha)\cdot\Delta\alpha$ corresponds to the relation of downwash and angle of attack. In a larger distance behind the lifting wing with quasi-elliptical wing-shape it can be assumed that

$$\frac{\partial\alpha_w}{\partial\alpha} \approx \frac{2}{\pi \cdot \Lambda} \cdot \frac{dc_{Lw}}{d\alpha} \qquad (6.1.15)$$

As shown in chapter 4, $c_{Lw,\alpha} = a_w \cdot a_{pw}(\alpha) \cdot 2 \cdot \pi$, wherein $a_{pw}(\alpha)$ takes care of the viscous airflow-effects in the boundary-layer of the airfoil used at the lifting wing. Thus we can write

$$\frac{\partial\alpha_w}{\partial\alpha} = \frac{4 \cdot a_w \cdot a_{pw}(\alpha)}{\Lambda} = \frac{4 \cdot a_w^{\times}}{\Lambda} \qquad (6.1.16)$$

For the case of non-stationary airflow *La Place*-transformation of the downwash-changes at the elevator yields with p=1/t

$$\Delta\alpha_w(p) = \frac{\partial\alpha_w}{\partial\alpha}\cdot\Delta\alpha(p)\cdot e^{-p\cdot\Delta t} \tag{6.1.17}$$

and for the effective increment of the angle of attack at the elevator follows

$$\Delta\alpha_h(p) = \Delta\alpha(p)\cdot(1-\frac{\partial\alpha_w}{\partial\alpha}\cdot e^{-p\cdot\Delta t}) \tag{6.1.18}$$

Development into a progression for small pitching-frequencies, $|p|<<1/\Delta t$ , will supply

$$\Delta\alpha_h(p) \approx \Delta\alpha(p)\cdot(1-\frac{\partial\alpha_w}{\partial\alpha})+\frac{\partial\alpha_w}{\partial\alpha}\cdot p\cdot\dot{\alpha} \tag{6.1.19}$$

Herewith for the change of the lift at the elevator results to

$$\Delta L_h = (c_{l,\alpha})_h\cdot\Delta\alpha_h\cdot q_h\cdot A_h \tag{6.1.19a}$$

$$\Delta c_{lh} = \frac{q_h}{q}\cdot\frac{A_h}{A}\cdot(c_{l,\alpha})_h\cdot\left[\Delta\alpha\cdot(1-\frac{\partial\alpha_w}{\partial\alpha})+\frac{\partial\alpha_w}{\partial\alpha}\cdot\Delta t\cdot\dot{\alpha}\right] \tag{6.1.19b}$$

The first term in equation 6.1.19b corresponds to the stationary change, the second term corresponds to an $\dot{\alpha}$ –derivative, and taking into account the definition

$$c_{L,\dot{\alpha}} \equiv \partial c_L/\partial(\dot{\alpha}\cdot\hat{c}/V)$$

we will get

$$c_{L,\dot{\alpha}} = \frac{q_h}{q}\cdot\frac{A_h}{A}\cdot\frac{\partial\alpha_w}{\partial\alpha}\cdot(c_{l,\alpha})_h\cdot\Delta t\cdot\frac{V}{\bar{c}} \tag{6.1.20}$$

$\Delta t$ can be calculated by means of the speed V and the path $r_h*$ which the changed wake has to travel from the lifting wing to the elevator, namely $\Delta t \approx r_h*/V$. Herewith for the $\dot{\alpha}$-derivative turns to be

$$c_{L,\dot{\alpha}} = \frac{q_h}{q}\cdot\frac{A_h}{A}\cdot\frac{r_h^\times}{\hat{c}}\cdot\frac{\partial\alpha_w}{\partial\alpha}\cdot(c_{l,\alpha})_h \tag{6.1.21}$$

The momentum-derivative

$$c_{m,\dot{\alpha}} = \partial c_m/\partial(\dot{\alpha}\cdot\hat{c}/V)$$

by means of the equation

$$\Delta M = -\Delta L_h\cdot r_h = \Delta L_h\cdot(r_h^\times-(X_{c.g.}-X_{Nw})) \tag{6.1.22}$$

can be written in the form

$$\boxed{c_{m,\dot{\alpha}} = -\frac{q_h}{q}\cdot\frac{A_h}{A}\cdot\frac{r_h^{\times 2}}{\hat{c}^2}\cdot(1-\frac{X_{c.g.}-X_{Nw}}{r_h^\times})\cdot\frac{\partial\alpha_w}{\partial\alpha}\cdot(c_{l,\alpha})_h} \tag{6.1.23}$$

The position of the aerodynamic centre of the wing, $X_{Nw}$ , depends on the lift- and momentum-derivatives of the chosen airfoils according to

$$\boxed{\frac{X_{Nw}}{\bar{c}} = \frac{1}{4}-\frac{c_{m,\alpha}}{c_{Lw,\alpha}}} \tag{6.1.24}$$

wherein $c_{Lw,\boldsymbol{\alpha}} = a_w\cdot a_{pw}(\alpha)\cdot 2\cdot\pi$.

For standard gliders $r_h^* \approx r_h$ and $q_h \approx q$. and thus with sufficient accuracy we can write

$$c_{m,\alpha} \approx -\frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (1 - \frac{X_{c.g.} - X_{Nw}}{r_h}) \cdot \frac{\partial \alpha_w}{\partial \alpha} \cdot (c_{l,\alpha})_h \qquad (6.1.24)$$

According to chapter 4, $c_{lh,\alpha} = a_h \cdot a_{ph}(\alpha) \cdot 2 \cdot \pi$, $a^x = a_w \cdot a_p(\alpha)$, and finally the attenuation derivative due to $\dot{\alpha}$ turns out to be

$$c_{m,\dot{\alpha}} \approx -2\pi \cdot a_h^{\times} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (1 - \frac{X_{c.g.} - X_{Nw}}{r_h}) \cdot \frac{\partial \alpha_w}{\partial \alpha} \qquad (6.1.25)$$

Even at major changes of $X_{Nw}$ due to viscous airfoil-effects for most standard-gliders $|X_{c.g.} - X_{Nw}| << r_h$ for the non-critical **α**-region of the wing-sections therefore with sufficient accuracy we can assume that

$$c_{m,\dot{\alpha}} \approx -2\pi \cdot a_h^{\times} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot \frac{\partial \alpha_w}{\partial \alpha} \qquad (6.1.25)$$

Altogether the attenuation-derivatives will supply

$$c_{m,\omega y} + c_{m,\dot{\alpha}} \approx -2\pi \cdot a_h^{\times} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \qquad (6.1.26)$$

According to formula 6.1.16 $\partial\alpha_w/\partial\alpha$ is of the order of magnitude of $4 \cdot a_w^* / \Lambda_w$, and consequently for gliders with higher aspect ratios of the lifting wing the downwash-derivative $\partial\alpha_w/\partial\alpha$ may be neglected without major error. Thus, finally it can be stated that the major contribution to the attenuation of the rotational movement of a glider results from the q-derivative.

## 6.1.c The α - derivative of $c_m$

The lift-dependence of a glider on the angle of attack within the non-critical **α**-range of the chosen airfoils in a first approach is composed of shares from the lifting wing and the elevator. The influence of the fuselage shall here be neglected. Since effects resulting from drag are also of secondary importance, from chapter 2 and with $\cos\alpha_w \approx 1$ and $|D_h \cdot \sin\alpha_w| << |L_h \cdot \cos\alpha_w|$ the total lift of the glider is given by

$$L = L_w + L_h , \qquad (6.1.27)$$

and using the manner of writing with lift coefficients

$$L = c_L \cdot q \cdot A, \qquad L_w = c_{Lw} \cdot q \cdot A_w, \qquad L_h = c_{lh} \cdot q \cdot A_h$$

We get

$$c_L = c_{Lw} + \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot c_{lh} \qquad (6.1.28)$$

Taking into account the downwash factor $\partial\alpha_w/\partial\alpha$ **,** the overall lift slope of the glider results to

$$c_{L,\alpha} = (c_{L,\alpha})_w + (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot (c_{l,\alpha})_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \qquad (6.1.29)$$

By use of formula 6.1.29 the α-derivative of the pitching-moment turns out to be

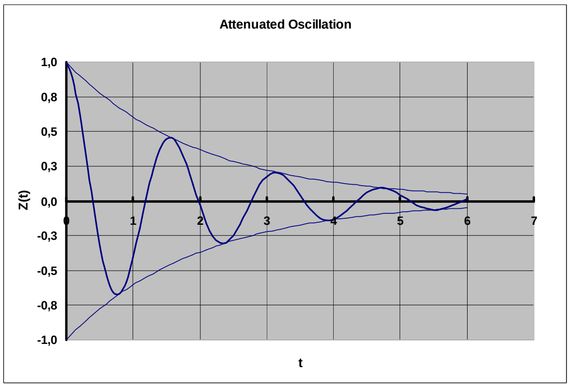
$$c_{m,\alpha} = (c_{L,\alpha})_w \cdot \frac{X_{c.g.} - X_{Nw}}{\hat{c}} + (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot (c_{l,\alpha})_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \qquad (6.1.30)$$

This derivative essentially depends on the size of the elevator and its distance from c.g. The position of the aerodynamic centre of the wing is influenced by the viscous effects of the chosen airfoils and given by equation 6.1.24.

By use of equations 4.15 and 4.16 and under the assumption that $q_h \approx q$ it finally follows

$$c_{m,\alpha} = 2\pi \cdot a_w^{\times} \frac{X_{c.g.} - X_{Nw}}{\hat{c}} + (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot 2\pi \cdot a_h^{\times} \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \qquad (6.1.31)$$

## 6.1.d Consequences for the Fast Pitching-Oscillations




Generally the characteristic equation of an attenuated oscillation is written in the form

$$\lambda^2 + 2 \cdot \delta \cdot \lambda + \omega_o^2 = 0 \qquad (6.1.32)$$

there in $\omega_o$ [$s^{-1}$] is called circular "eigen"-frequency and $\delta$ [$s^{-1}$] is called attenuation constant. Attenuated oscillation is given for the case when $\delta < \omega_o$.

In this case equation 6.1.32 has two conjugated complex solutions

$$\lambda_{1;2} = -\delta \pm j\,\omega \quad \text{with} \quad \omega = \omega_o^2 - \delta^2$$ ( $j$ denotes the imaginary unit).

A disturbance Z(t) then results from the general solution

$$Z(t) = (C_1 \cdot e^{j\omega t} + C_2 \cdot e^{-j\omega t}) \cdot e^{-\delta \cdot t} \qquad (6.1.33a)$$

At t = 0, $Z = Z_o = Z(t_o)$ for the undetermined coefficients $C_1$ and $C_2$ results $Z_o = C_1 + C_2$, and for our purposes Z(t) can finally be transformed into equation

$$Z(t) = Z(t_o) \cdot \delta^{-\delta \cdot t} \cdot \cos(\omega \cdot t + \varphi) \qquad (6.1.33)$$

Above graphic illustrates the attenuation of a disturbance Z(t) with time, the two enveloping curves describe the time dependent damping of the oscillatory motion after the disturbance.

The larger the attenuation constant $\delta$, the more rapidly the envelope $Z(t_o) \cdot \exp(-\delta \cdot t)$ approaches 0. The usual measure for it is $D = \delta / \omega_o$.

By comparison of equation 6.1.2 with equation 6.1.32, for the attenuated oscillations of the pitching movement of the glider after disturbance of the stationary gliding we get

$$\delta = -\frac{1}{2 \cdot J_y} \cdot \frac{\hat{c}}{V} \cdot (c_{m,\alpha} + c_{m,\omega y}) \cdot q \cdot A \cdot \hat{c} \qquad (6.1.34)$$

Taking into account equation 6.1.26 we will finally get

$$\delta = \frac{\pi}{2 J_y} \cdot a_h^{\times} \cdot A_h \cdot r_h^2 \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \cdot \rho \cdot V \qquad (6.1.35)$$

Here $\rho$ is the density of the air.

- This equation tells us that after disturbance of the angle of attack and/or the gliding angle, the damping of pitching-oscillations mainly depends on shape and geometry of the elevator and in particular most strongly on the distance of the elevator from c.g. since this acts with the second power.

- In the non-critical $\alpha$-range usually the viscosity factor of the elevator airfoil $a_{ph}(\alpha)\geq 1$, in particular at very low Re-numbers where viscosity-effects in the boundary layer of the airfoil play a considerable role for the airflow. In order to make sure that a glider will provide desired attenuation, the lower limit $a_{ph}=1$ should be chosen in equation 6.1.35, and correspondingly the other parameters for the required $\delta$.

- As mentioned earlier, regard to the downwash is paid by $\partial\alpha_w/\partial\alpha \approx 4\cdot a_w^{\times}/\Lambda_w$. In principle it will become smaller with increasing aspect-ratio of the lifting wing, and as will still be discussed later, due to deteriorating viscous-effects with increasing flight velocity it will decrease with increase of the velocity. At lower velocity, for gliders with small aspect-ratio ($\Lambda_w \approx 10$) the downwash factor may become $\partial\alpha_w/\partial\alpha \approx 0.5$, at higher speed for gliders with higher aspect ratio (($\Lambda_w \approx 25$) the lower border will be in the range of $\partial\alpha_w/\partial\alpha \approx 0.15$. This means that the attenuation of the pitching-oscillation will become smaller with increasing aspect-ratio of the lifting wing which has to be taken into account for the size and the momentum-arm of the elevator.

- We also learn from equation 6.1.35 that the attenuation of the pitching oscillation increases with the speed and with the mean aerodynamic chord (MAC) of the glider.

- A factor to which most often not sufficient attention is drawn is the mass-moment of inertia $J_y$ around the lateral axis of the glider. According to equation 3.4. the masses of the tail and the nose of the glider contribute most to this moment, thus in order to achieve proper attenuation and to keep the elevator dimensions small, according to equation 6.1.35 a construction goal should be to keep the elevator mass as low as possible (correspondingly the mass in the front part of the fuselage can be reduced).

- The attenuation of the pitching oscillation, however, must not be chosen too strong because on the other side the response to the elevator control-panel may become too slow for the necessary manoeuvrability. When designing a new glider mostly it can be very helpful to determine the values of the characteristic parameters for the attenuation from gliders known to provide the required $\delta$-measure.

For the circular "eigen"-frequency $\omega_o$ of the corresponding non-attenuated oscillation it matters

$$\boxed{\omega_o^2 = -\frac{1}{J_y}\cdot c_{m,\alpha}\cdot q\cdot A\cdot \hat{c}} \qquad (6.1.36)$$

and taking into account equation 6.1.31 it will turn out to become

$$\boxed{\omega_o^2 = -\frac{1}{J_y}\cdot\left(2\pi\cdot a_w^{\times}\frac{X_{c.g.}-X_{Nw}}{\hat{c}} - (1-\frac{\partial\alpha_w}{\partial\alpha})\cdot 2\pi\cdot a_h^{\times}\cdot\frac{q_h}{q}\cdot\frac{A_h}{A}\cdot\frac{r_h}{\hat{c}}\right)\cdot q\cdot A\cdot\hat{c}} \qquad (6.1.37)$$

- Since the aerodynamic centre of the lifting wing is determined by the wing-design and the chosen airfoils and the position of the c.g in principle results from the requirements for optimum gliding and/or minimum sinkrate, and since all other parameters are determined by the requirement for sufficient static stability and attenuation, there is no more possibility to affect $\omega_o$.

## 6.2 Slow Oscillations of the Centre of Gravity

At instationary longitudinal motion of a glider with constant angle of attack, $\Delta\alpha \equiv 0$, according to *F.W. Lancaster* a so called "pitch-phugoid" develops after disturbance in V and/or $\vartheta$. This usually is a long-period mode in which the c.g. carries out a lightly damped oscillation about its stationary flight path. It involves a slow pitching-oscillation over many seconds in which energy is exchanged between vertical and forward velocity. (The equations of motion now just provide information on the angle of the elevator control necessary to maintain a constant angle of attack.)

The relations between the weight $G = m\cdot g$, the lift L, the drag D of the plane, and the gliding angle $\vartheta$ are to be derived from equations 6.1 and 6.2 of the forces in the directions of the x- and z-axes. When setting $\Delta\alpha \equiv 0$ we receive

$$m \cdot \dot{V} = X_V \cdot \Delta V + X_\vartheta \cdot \Delta\vartheta \qquad (6.2.1)$$

$$m \cdot V \cdot \dot{\vartheta} = Z_V \cdot \Delta V + Z_\vartheta \cdot \Delta\vartheta \qquad (6.2.2)$$

$$0 \equiv \begin{vmatrix} m \cdot \lambda - X_V & -X_\vartheta \\ -Z_V & m \cdot V \cdot \lambda - Z_\vartheta \end{vmatrix} \qquad (6.2.3)$$

Thereupon the characteristic equation of the pitch-phugoid can be written in the form

$$m^2 \cdot V \cdot \lambda^2 + (-m \cdot G \cdot \sin\vartheta + \frac{\partial D}{\partial V} \cdot m \cdot V) \cdot \lambda + (-\frac{\partial D}{\partial V} \cdot G \cdot \sin\vartheta + \frac{\partial L}{\partial V} \cdot G \cdot \cos\vartheta) = 0 \qquad (6.2.4)$$

Since $L = L(V^2)$ and $D = D(V^2)$, it follows

$$\partial L/\partial V = 2 \cdot L/V \quad \text{and} \quad \partial D/\partial V = 2 \cdot D/V$$

and with $L = G \cdot \cos\vartheta$ , $D = G \cdot \sin\vartheta$ we get

$$m^2 \cdot V \cdot \lambda^2 + (-m \cdot G \cdot \sin\vartheta + 2 \cdot m \cdot G \cdot \sin\vartheta) \cdot \lambda + (-\frac{2}{V} \cdot G^2 \cdot \sin^2\vartheta + \frac{2}{V} \cdot G^2 \cdot \cos^2\vartheta) = 0 \qquad (6.2.5)$$

Thus finally the characteristic equation can be written in the form

$$\lambda^2 + \frac{g \cdot \sin\vartheta}{V} \cdot \lambda + \frac{2 \cdot g^2}{V^2} \cdot (\cos^2\vartheta - \sin^2\vartheta) = 0 \qquad (6.2.6)$$

For smaller gliding angles $\sin^2\vartheta \approx 0$.

## 6.2.a Consequences for the slow c.g.-oscillations

Like for the fast pitching oscillations, the general characteristic equation of the damped c.g.-oscillation is to be written in the form

$$\lambda^2 + 2 \cdot \delta \cdot \lambda + \omega_o^2 = 0 \qquad (6.2.7)$$

Therein $\omega_o$ [$t^{-1}$] is called circular "eigen"-frequency and $\delta$ [$t^{-1}$] is the damping constant. Attenuated oscillation is given for the case when $\boldsymbol{\delta < \omega_o}$. Like for the characteristic equation of the fast pitch oscillation, then there will exist two solutions $\lambda_3$ and $\lambda_4$

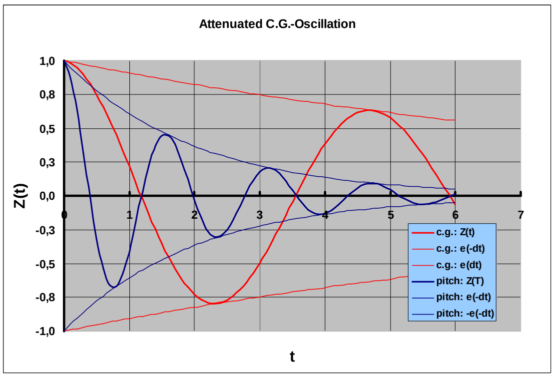
$$\lambda_{3;4} = -\delta \pm j\,\omega \quad \text{with} \quad \omega = \omega_o^2 - \delta^2$$

This again leads to a damped oscillating disturbance. Comparison of equation 6.2.7 with equation 6.2.6 yields:

Damping constant: $$\delta = \frac{g \cdot \sin\vartheta}{2 \cdot V} \qquad (6.2.8)$$

"Eigen"-frequency: $$\omega_o = \frac{g}{V} \cdot \sqrt{2 \cdot (\cos^2\vartheta - \sin^2\vartheta)} \approx \sqrt{2} \cdot \frac{g \cdot \cos\vartheta}{V} \qquad (6.2.9)$$

Thus, damping and “eigen”-frequency only depend on the velocity V and on the gliding angle $\vartheta$ of the corresponding stationary flight-state. They do not depend on the characteristics of a given glider.




The left graphic provides a rough idea of the difference between the fast attenuated pitching-oscillations and the slow, damped c.g. oscillations.

The subsequent examples will provide the relations as they are observed in flying practice.

## 6.3 Coupled Pitch- and C.g.-Oscillations

In some cases it may be desired to consider the equations of motion for a concurrent disturbance in velocity, gliding angle and angle of attack. In this case the characteristic equation $F_4(\lambda)$ (equation 6.5 and 6.6)) has to be solved. The coupling of fast pitch- and slow c.g.-oscillations will cause a certain shift of the roots $\lambda_1$ to $\lambda_4$. As before $\lambda_{1;2}$ may be the roots of the fast pitching-motion and $\lambda_{3;4}$ those of the slow phugoid-motion. Using the designation of the coefficients of $F_4$ as in equation 6.6 the mathematic evaluation provides following equations for the four roots:

$$\lambda_1 + \lambda_2 = -B - (\lambda_3 + \lambda_4) \qquad (6.3.1a)$$

$$\lambda_1 \cdot \lambda_2 = C - \lambda_3 \cdot \lambda_4 - (\lambda_1 + \lambda_2) \cdot (\lambda_3 + \lambda_4) \qquad (6..3.1b)$$

$$\lambda_3 \cdot \lambda_4 = E / (\lambda_1 \cdot \lambda_2) \qquad (6.3.1c)$$

$$\lambda_3 + \lambda_4 = -\,(D + (\lambda_1 + \lambda_2) \cdot \lambda_3 \cdot \lambda_4\,) / (\lambda_1 \cdot \lambda_2) \qquad (6.3.1d)$$

For the initial approximation

$$(\lambda_3 + \lambda_4)^{(0)} = 0 \qquad \text{and} \qquad (\lambda_3 \cdot \lambda_4)^{(0)} = 0$$

as a first solution is yielded:

$$(\lambda_1 + \lambda_2)^{(1)} = -B \qquad (6.3.2a)$$

$$(\lambda_1 \cdot \lambda_2)^{(1)} = C \qquad (6.3.2b)$$

$$(\lambda_3 \cdot \lambda_4)^{(1)} = E / C \qquad (6.3.2.c)$$

$$(\lambda_3 + \lambda_4)^{(1)} = (-D \cdot C + B \cdot E) / C2 \qquad (6.3.2.d)$$

In a second step then the roots $\lambda_1$ to $\lambda_4$ can be determined and the corresponding motion investigated. This will not further be followed up in this context.

For RC-controlled planes it is rather important that the fast pitching-oscillation is sufficiently damped since otherwise the pilot will not be able to correct these disturbances. On the other hand, to correct slow phugoidal oscillations does usually not cause any problems. According to experience for most standard gliders the damping of the pitch-disturbances is such that the flight behaviour after disturbances can be predicted by means of the separated characteristic equations of motion for fast pitching-oscillations and slow phugoidal movement. It may be important to solve the coupled characteristic equation for “wing only”-gliders with minor degree of longitudinal attenuation in order to predict their flight stability.

## 7. Examples of Proven Gliders

### 7.1 Assessment of the Mass Moment of Inertia, $J_y$

According to chapter 3 the mass moment of inertia is given by the equation

$$J_y = \sum_i m_i \cdot r_i^2$$

Inertia forces derive from the attribute of the mass to resist accelerations. The mass of rotational accelerations is represented by mass moment of inertia terms J. The total mass moment of inertia related to the rotation of a glider around the lateral y-axis through the c.g., Jy, results from the various parts of the glider: the lifting wing, the fuselage, and the tail-parts (fin and elevator, or V-tail).

An approximate value of $J_y$ can be assessed for most gliders according to the approach

$$J_y = m_w \cdot r_w^2 + m_{f,f} \cdot r_{f,l}^2 + m_{f,r} \cdot r_{f,r}^2 + m_t \cdot r_t^2 \qquad (7.1.1)$$

Therein $m_w$ denotes the mass of the lifting wing, $r_w$ the distance of the c.g. from the mass centre of the wing, $m_{f,f}$ the share of the fuselage-mass in front of the c.g., $r_{f,f}$ the distance of the c.g. from the mass-centre in the front of the fuselage, $m_{f,r}$ the mass of the rear-tube of the fuselage behind the c.g., $r_{f,r}$ the distance of the c.g. from the mass-centre of the rear-fuselage part, $m_t$ the mass of the tail and $r_t$ the distance of the c.g. from the mass-centre of the tail.

Later on two examples will be given. One of them will be that of an F3J-glider with 3.7 meter wingspan and a mass of approximately 2.3 kg. For this model it was theoretically estimated that

$m_w$ = 1.30 kg, $r_w$ = 0.03 meter

$m_{f,f}$ = 0.68 kg, $r_{f,f}$ = 0.4 meter

$m_{f,r}$ = 0.28 kg, $r_{f,r}$ = 0.7 meter

$m_t$ = 0.12 kg, $r_t$ = 1.15 meter

Herewith the mass-moment of inertia was expected to become

$$J_y = 1.30 \cdot 0.03^2 + 0.68 \cdot 0.4^2 + 0.2 \cdot 0.7^2 + 0.12 \cdot 1.15^2 \ \text{kg} \cdot \text{m}^2$$

$$= 0.0012 \quad + 0.109 \quad + 0.098 \quad + 0.159 \ \text{kg} \cdot \text{m}^2$$

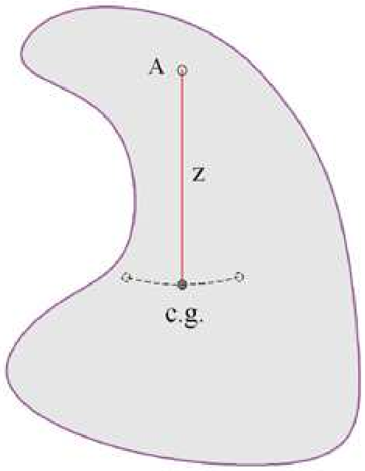
$$= 0.367 \ \text{kg} \cdot \text{m}^2$$

We see that the smallest contribution results from the lifting wing because its mass-centre is rather close to the c.g., whilst the largest contribution results from the tail-part which has the lowest mass, but its distance from the c.g. is the largest.

Generally, in order to keep the mass-moment of inertia small as desired by the damping-requirements, at standard-gliders the weight of the tail should be kept as low as possible. Each gram saved at the tail also reduces the balancing-ballast in the nose of the fuselage about factor 2 to 3 and correspondingly also the mass-moment of inertia of the fuselage-front.

- As was shown in chapter 6, $J_y$ plays an important roll for the attenuation of the fast pitching oscillations, see equation 6.1.35. With increasing value of $J_y$ in general the size $A_h$ or the momentum arm $r_h$ of the elevator have to be increased to compensate for. If also a certain static stability is required according to chapter 4, equation 4.1.7., the right balance between $A_h$ and $r_h$ has to be found.

## 7.2 Experimental Determination of the Mass -Moment of Inertia, $J_y$




A simple practical method to determine the mass-moment of inertia of any given body is as follows. If a body like that in the left graphic is suspended with an axis through A it can be stimulated to swing around this axis and the time T for one full period of oscillations is given by

$$T \approx 2 \cdot \pi \cdot \sqrt{J/m \cdot g \cdot z} \qquad (7.2.1)$$

Herein J is the mass-moment of inertia related to the axis through A, z is the distance from the c.g. According to physical mechanics J can also be described in the form

$$J = J_{c.g.} + m \cdot z^2 \qquad (7.2.2)$$

where $J_{c.g.}$ is the mass-moment of inertia for the body related to the axis through the centre of gravity, parallel to the axis through A.

Combining the two equations we get

$$J_{c.g.} \approx \left( \frac{T}{2 \cdot \pi} \right)^2 \cdot m \cdot g \cdot z - m \cdot z^2 \qquad (7.2.3)$$

$\omega \approx 2\pi/T$ is the oscillation-frequency of this swing of pendulum.

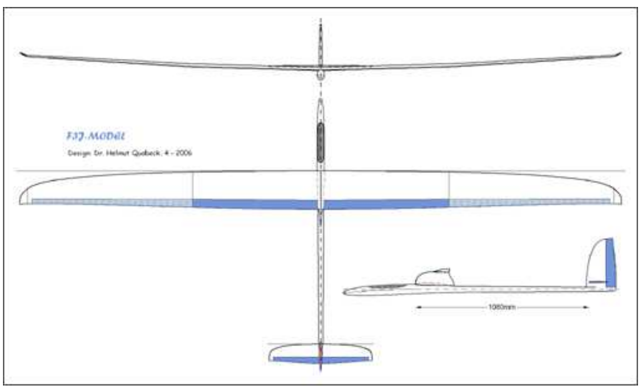
By means of this pendulum-method for a given model-plane the mass moment of inertia $J_y$ around the lateral y-axis through the c.g. can easily be determined.

For example, when the F3J-Model given in section 7.2 was hung up with nose down at the end of the fuselage it swung with a period-time T = 2.32 s. With a distance of the swinging-axis from the c.g., z = 1.2 meters, by means of formula 7.2.3 this yields

$$J_y = (2.32/2\pi)^2 \cdot 2.3 \cdot 9.81 \cdot 1.2 - 2.3 \cdot 1.2^2 = 0.38 \text{ kg}\cdot\text{m}^2$$

The above theoretical estimate of 0.367 $\text{kg}\cdot\text{m}^2$ differs not much from the practical result. In order to determine the appropriate values of the geometric parameters of the model for proper static stability and attenuation of the fast pitching oscillations it was a good guide.

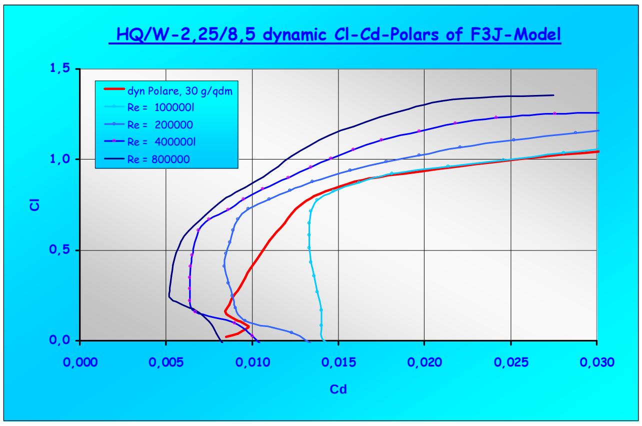
## 7.3 Example of an F3J-Model



The left graphic shows the 3 side-draft for a new F3J-model planned by the author. Flight-mechanical characteristics of the model as given below have been determined by means of the *"FMFM"*-program (*Flight-Mechanics for Flight-Models*) which is described in more detail on the homepage www.hq-modellflug.de.

Major goals for the model were superior sinkrates and gliding-performance at all flight conditions, as well as proper flight-stability and manoeuvrability as required in F3J-contests.

a. Since the lifting wing is mainly responsible for the performance of a glider-model, major attention has been turned to its geometric outlay and its aerodynamic characteristics such as lift-efficiency, airfoil-and induced drag. The airfoils finally chosen are the *“HQ/W-2.25/8.5”* strait through for the lifting wing, and the *“HQ/W-0/9* for elevator and fin. The distribution of the wing chord was chosen such that the lift-distribution of the model is close to ideal. According to good practical experience, stall problems at the lifting wing can be handled by appropriate wingtips.




b. Usually the selection of distinct airfoils for a lifting wing is done by a comparison of the performance of potential airfoils over the possible speed range given by the weight/unit area. For manned gliders this at the end is given in the form of a quasi-stationary velocity polar. In the first instance it requires that for all possible stationary velocities of the glider the corresponding lift, the airfoil- and the induced drag must be determined for the lifting wing. In the left polar-graphic of the *“HQ/W-2,25/8,5”*-profil this particular quasi-stationary polar is indicated by the red polar curve.

Other $c_l$-$c_d$-values then those on the red quasi-stationary polar may be reached under instationary flight conditions, such as given at a fast turn or a loop, however, this is of minor importance for the performance-considerations.

The stationary gliding- and sink-velocities of a glider are given by

$$V = \sqrt{\frac{2}{\rho} \cdot \frac{m \cdot g}{A} \cdot \frac{\cos\vartheta}{c_L}} \approx 4 \cdot \sqrt{\frac{m}{A} \cdot \frac{\cos\vartheta}{c_L}} \qquad (7.3.1)$$

$$V_z = \sqrt{\frac{2}{\rho} \cdot \frac{m \cdot g}{A} \cdot \frac{c_W^2}{c_L^3} \cdot \cos^3\vartheta} \approx 4 \cdot \sqrt{\frac{m}{A} \cdot \frac{c_W^2}{c_L^3} \cdot \cos^3\vartheta} \qquad (7.3.2)$$

The corresponding stationary gliding number is given by

$$G.N. = 1/\tan\vartheta = L / D = c_L / c_D \qquad (7.3.3)$$

In order to determine the potential performance of a given lifting wing with chosen wing-sections the author usually ascertains the functional dependence of the wing only sinkrates and gliding numbers given by

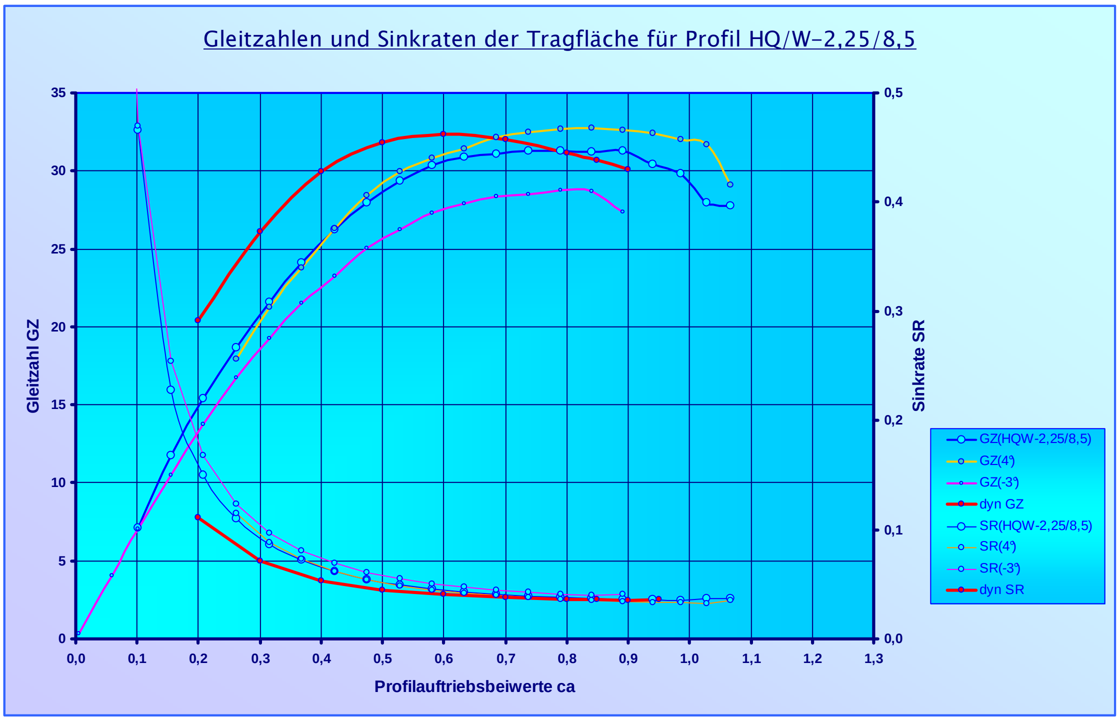
$$S.R._w = c_{Dw}^2 / c_{Lw}^3 \qquad (7.3.4)$$

$$G.N._w = c_{Lw} / c_{Dw} \qquad (7.3.5)$$

If only one airfoil is chosen without twist, like for the F3J-model, then $c_{Lw} = a_w \cdot c_l$, where $a_w$ denotes the lift-efficiency-factor of the wing (which can easily be determined by the *FMFM*-program) and $c_l$ is the lift coefficient of the chosen airfoil. In other cases $c_{Lw}$ must be determined by integration which will not further be explained here (e.g. such a method is included in the *FMFM*-program).

The drag related to the chosen lifting-coefficient results from the properties of the airfoil and from the free vortices, in total we have $c_{Dw} = c_{Dp} + c_{Di} \approx c_{Dp} + c_{Lw}^2/\pi\Lambda_w$.

Correspondingly for the F3J-model under consideration the graphic below reflects the dependence of the gliding number G.N. and of the sinkrate S.R. on the velocity, indirectly given by the $c_l$ of the airfoil. In this chart are also included the G.N.- and S.R.-curves for flap deflection. By means of such a chart the optimum $c_{Lw}$-$c_{Dw}$-working point can easily be identified either for minimum sinking or optimum gliding as required at slow stationary flying in F3J-contests.




For the planned F3J-model the optimum working point is around $c_l$ = 0.9. Next we will see where the c.g. must be located in order to achieve this working point whilst flying.

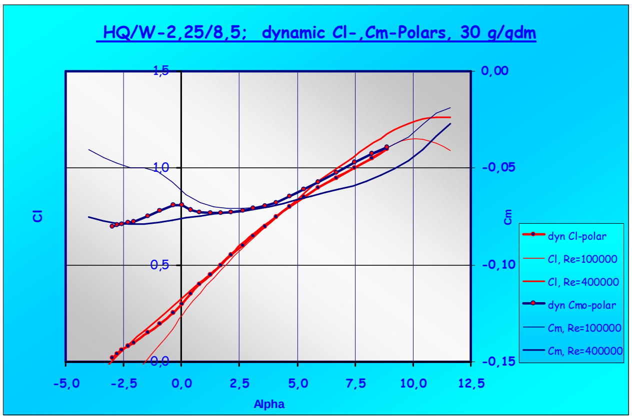
c. Having determined the optimum $c_{Lw}$-$c_{Dw}$-working point of the lifting wing for slow performance gliding, next the position of the centre of gravity c.g. which enables the glider to achieve these optimum flight conditions at soaring has to be found. According to flight dynamics, with $c_{Mf}$ denoting the momentum-coefficient of the fuselage, and assuming that the momentum coefficient of the elevator can be neglected, the longitudinal momentum equation for the centre of gravity generally yields.

$$\frac{X_{c.g.}}{\hat{c}} = \frac{X_{Nw}}{\hat{c}} + \frac{1}{c_{Lw}}\left( c_{Lh} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} - c_{Mow} - c_{Mf} \right) \qquad (7.3.6.a)$$

For the thinner F3J-fuselages $c_{Mf}$ can be neglected, thus for zero lift at the elevator it turns out

$$\frac{X_{c.g.}}{\hat{c}} = \frac{X_{Nw}}{\hat{c}} - \frac{c_{Mow}}{c_{Lw}} \qquad (7.3.6.b)$$

In cases like the one being considered where the same profile is used in all wing sections, the momentum coefficient $c_{Mow}$ corresponds to that of the profile and the lift coefficient $c_{Lw}$ is given by $c_{Lw} = a_w \cdot c_l$, where $c_l$ is the lift coefficient of the profile. For example the coefficients $c_{mo}$ and $c_l$ are taken for the *“HQ/W-2,25/8.5”*-airfoil from the corresponding *X-FOIL*-polar-diagrams shown below.




In this graphic in particular the quasi-stationary polars of the coefficients for lift (thick red curve) and the airfoil momentum (thick dark blue line) are given for a wing load of 3.0 kg/m$^2$. By means of the *FMFM*-Program the lift efficiency-factor of the wing was calculated to be

$a_w$ = 0.897.

From the above chart the momentum coefficient corresponding to the lift coefficient for optimum performance, $c_l = 0.9$, was taken to be

$$c_{Mow} = -0{,}059.$$

Thus we get

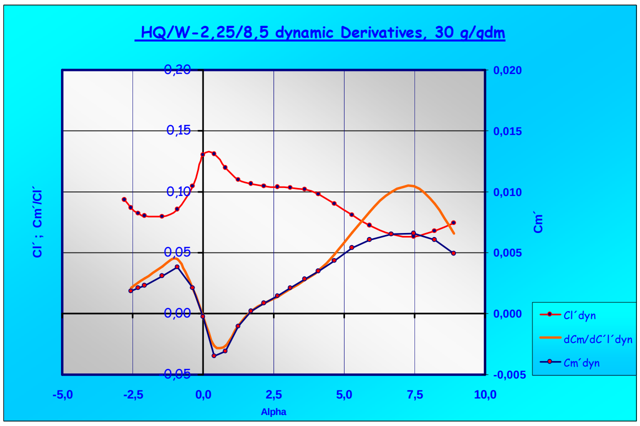
$$X_{c.g.}/\hat{c} = X_{Nw}/\hat{c} - (-0.059/(0.897 \cdot 0.9)) = X_{Nw}/\hat{c} + 0.073$$

If no viscous effects would influence the airstream at the profile the aerodynamic centre of the wing would be at 25 % and the c.g. would then be at 32.3 % of the MAC. As can be seen in the polar chart, according to the *X-Foil*-analyses the lift- and momentum-coefficients for a given Reynold-number, e.g. Re=100000 and Re=400000, do not behave linear, even not in the non-critical α-range, but approach linear performance with increasing Re-numbers.

As already pointed out in chapter 6, due to the viscous airstream at the airfoil the aerodynamic centre of the wing shifts away from the ideal 25 % position according to formula 6.1.24

$$\frac{X_{Nw}}{\hat{c}} = \frac{1}{4} - \frac{c_{m,\alpha}}{c_{Lw,\alpha}}$$

The quasi-stationary development of the profile derivatives $c_{m,\alpha}$ and $c_{l,\alpha}$ as functions of the angle of attack α is shown in the following graphic. The curves are traced back to the corresponding curves in the previous graphic. The thinner lines represent the dependence of the derivatives on the angle of attack and the thick red line their ratio.




As mentioned earlier, $c_{Lw,\alpha} = a_w \cdot c_{l,\alpha}$ in case of a lifting wing with uniform airfoil.

Thus with the derivatives taken from the left chart for $c_l$=0.9, for the planned F3J-model the theoretical aerodynamic centre turns out to be

$$\frac{X_{Nw}}{\hat{c}} = \frac{1}{4} - \frac{c_{m,\alpha}}{a_w \cdot c_{l,\alpha}}$$
$$= 0.25 - 0.065 / 0.897$$
$$= 0{,}178$$

And consequently the position of the centre of gravity should be chosen at

$$X_{c.g.}/\hat{c} = 0.178 + 0.073 = 0.251$$

***Remarks and comparison to flight experience***

*This c.g.-position appears to be rather far in front of the lifting wing; however, two aspects have to be considered:*

- *First, the chosen working point $c_l$ = 0.9 is rather high but the corresponding c.g. will allow to achieve all stationary flight conditions for all smaller $c_l$-values.*

- *Secondly, for exemplary reasons the aerodynamic coefficients and their derivatives for this particular F3J-case have been developed by means of the X-Foil-program of Mark Drela which to some degree seems to overemphasise the viscous airstream effects. The Profile-program of*

*Richard Eppler on the other hand appears to underestimate them and would have yielded a position for the aerodynamic centre closer to the quarter-point of the MAC.*

- *As the long term practical experience of the author with many different models has shown, in most cases the aerodynamic centre of the lifting wings is closer to the 25 % position then to that resulting from the X-Foil-program! But, fortunately, in practice a bad position of the c.g. will soon be found out and corrected.*

- *As has been shown in previous chapters, the c.g.-position is the point of reference for the static and dynamic stability of the glider. If the aerodynamic centre of the wing is assumed too far in front of the wing and consequently also the c.g. it may happen that the stability-characteristics such as the elevator size and the momentum arm may be chosen too small. This will further still be discussed in the next section.*

*E.g. for a working point $c_l = 0.8$ the position of the aerodynamic centre would turn out to be at 19.4 % and that of the c.g. at 28.2 %, the soaring performance would not much differ.*

d. Next step in the design-routine of a plane will usually be to determine the dimensions and aerodynamic features of the elements for longitudinal flight control and stability. For a normal glider these elements are the rear fuselage-part behind the c.g. and the elevator. (For wing only models S-shaped profiles, sweep, and negative twist of the lifting wing will take over the longitudinal stability functions, but this case will be discussed in another paper.)

The static stability for sure is most important for the longitudinal flight-stability of a plane, as laid out in chapter 5:

$$\sigma = (X_N - X_{c.g.}) / \hat{c}$$

From many glider constructions the author got the feedback, that the longitudinal stability of F3J-gliders should at least be 0.1:

$$\sigma \geq 0.1 \quad !$$

Consequently we have the requirement for the elevator and its momentum arm related to the c.g. that according to chapter 4 they have to be sized such that the aerodynamic centre of the total plane fulfils the requirement

$$X_N / \hat{c} \geq 0.1 + X_{c.g.} / \hat{c}$$

In chapter 4 the final equation for the dependence of the aerodynamic centre on the wing- and elevator-characteristics was

$$\frac{\Delta X_N}{\hat{c}} = \frac{a_w^\times \cdot a_h^\times \cdot A_h / A}{1 + a_w^\times \cdot a_h^\times \cdot A_h / A} \cdot \frac{r_{Nh}}{\hat{c}}$$

with $a_w^x = a_w \cdot a_{pw}(\alpha)$ and $a_h^x = a_h \cdot a_{ph}(\alpha)$.

From the last graphic it can be taken, that the $\alpha$-derivative of the profile-lift-coefficient for the lifting wing varies between $c_{l,\alpha} = 0.08\ \text{grad}^{-1}$ for the chosen working point $c_l = 0.9$ and $c_{l,\alpha} = 0{,}13\ \text{grad}^{-1}$ for low speed, while at medium angles of attack (1° - 4°) it shows values about $c_{l,\alpha} \approx 0.104\ \text{grad}^{-1}$. In the calculations here $\boldsymbol{\alpha}$ must be transformed to rad-units, ergo:

$$4.58 \leq c_{l,\alpha} \leq 7.44 \quad \text{rad}^{-1}$$

Since $a_p = c_{l,\alpha}/2\pi$, correspondingly we receive

$$0.73 \leq a_{pw} \leq 1.19,$$

For the range of more dynamic soaring $c_{l,\alpha} \approx 6$ grad$^{-1}$ which is not far from the ideal $2\pi$-value and $a_{pw} \approx 1$. As we have seen earlier in this chapter, the lift efficiency-factor of the chosen wingform is $a_w = 0.897$, and according to experience the lift-efficiency of the elevator is assumed to be about $a_h \approx 0.75$.

The major working-conditions of the elevator are about zero lift. The X-*Foil*-analyses of suitable airfoils for the elevator tell us, that for the lift range of the elevator $a_{ph} \approx 1,2$.

Based on these data and on a first crude estimate $A/A_h \approx 0.1$ for the ratio of the areas of elevator and lifting wing, and the simplification that approximately $r_{Nh} \approx r_h$ we receive

$$0.056 \cdot r_h / \hat{c} \leq X_N / \hat{c} \leq 0.088 \cdot r_h / \hat{c}$$

Taking the c.g. as found for the working point $c_l = 0.9$, namely $X_{c.g.} / \hat{c} = 0.251$, on the other hand side we have $X_N / \hat{c} = 0.15+0.251 = 0.4$. Consequently the range for the momentum-arm $r_h$ is found to be

$$6.3 \geq r_h / \hat{c} \geq 4.0,$$

e. In section 6.1.d it was derived that the attenuation-constant **δ** of fast pitching-oscillations is affected by various parameters as finally given in equations 6.1.34/35, written in a more practical form we get

$$\delta = -\frac{q \cdot A \cdot \hat{c}^2}{2 \cdot V} \cdot \frac{(c_{m,\dot{\alpha}} + c_{m,\omega y})}{J_y}$$

$$= -\frac{\rho \cdot V \cdot A \cdot \hat{c}^2}{4} \cdot \frac{1}{J_y} \cdot \left( -2 \cdot \pi \cdot a_h^{\times} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \right)$$

First we see that the damping of the oscillations increases with the soaring velocity V. Thus, minor influences due to inviscous airfoil effects which are reflected by the factors $a_{pw}$ and $a_{ph}$ are generally overwhelmed with increasing velocity.

Secondly, for a given wing the major parameters by which attenuation can be influenced are the mass-moment of inertia $J_y$ and size $A_h$, shape (factor $a_h$), and momentum arm $r_h$ of the elevator.

For a lifting wing the $\alpha$-derivative of the downwash far behind the wing is given by $\partial\alpha_w/\partial\alpha \approx 4 \cdot a_w^{\times} / \Lambda_w$. Thus, it decreases with the aspect-ratio $\Lambda_w$ of the lifting wing. With data given earlier the $\alpha$-derivative for the planned F3J-Glider ranges according to $0.15 \leq 4 \cdot a_w^{\times} / \Lambda_w \leq 0.25$ and cannot be influenced by the elevator characteristics.

Consequently the only remaining design-element for proper damping of disturbances is the ratio of the q-derivative and the mass-moment of Inertia

$$\boxed{\frac{c_{m,\omega y}}{J_y} = -\frac{1}{J_y} \cdot 2 \cdot \pi \cdot a_h^{\times} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2}}$$

From practical experience with various F3J-models and analyses of successful other F3J-models the author has found that appropriate dynamic damping is achieved when this ratio ranges within -30 to -40.

As was laid out in section 7.1, for a lower-weight F3J-model the mass-moment of inertia is around $J_y = 0.4$ kg·m$^2$. However, when a model is being build it may easily happen that the weight of the tail gets higher than desired and, since the mass of the tail contributes most to $J_y$, a minor damping than planned will appear. Thus, in order to be on the safe side concerning dynamic longitudinal damping, it may often be better to assume that $J_y \approx 0.5$. Since damping increases with flight-velocity, the viscous airstream-effects at the elevator are in fact only important for flight-conditions near the working point $c_l = 0.9$.

Assuming for the planned F3J-model as before that $a_h = 0.75$, $a_{ph} \approx 1.2$, $A_h/A \approx 0.1$, and $J_y = 0.367$ we yield the requirement

$$3.8 \leq r_h / \hat{c} \leq 5.1$$

f. For the F3J-model shown above the following model-parameters were chosen:

| | | |
|---|---|---|
| Mean chord | $\hat{c}$ = | 209.54 mm |
| Lifting-area of the wing | A = | 0.704 $m^2$ |
| Aspect ratio of the wing | $\Lambda_w$ = | 17.41 |
| Lift-efficiency of the wing | $a_w$ = | 0.897 |
| Airfoil of the wing | *HQ/W-2,25/8,5* | |
| Mean chord of the elevator | $\hat{c}_h$ = | 101.5 mm |
| Lifting-area of the elevator | $A_h$ = | 0.065 m2 |
| Lift-efficiency of the elevator | $a_h$ = | 0.76 |
| Airfoil of the elevator | *HQ/W-0/9* | |

For the most important flight state around the chosen working point $c_l = 0.9$ we assume

Airfoil efficiency-factor $a_{ph} \leq$ 1.2 (at zero elevator lift)

Further

Mass moment of inertia $J_y$ = 0.367 kg · $m^2$

There from dynamic stability is calculated to be:

| | | |
|---|---|---|
| Dynamic stability measure | $c_{m,\omega y}/ J_y \approx -33.9$ ! | |

This is well within the required stability range.

Taking into account the viscous effects of the airstream around the lifting wing (*X-FOIL*-analysis), above we had found that in order to achieve the working point conditions around $c_l = 0.9$ the position of the centre of gravity should be chosen at

| | | |
|---|---|---|
| Centre of gravity | $X_{c.g.}/\hat{c}$ = | 0.251 |

Then the length of the momentum arm between c.g. and the aerodynamic centre of the elevator becomes

| | | |
|---|---|---|
| Length of momentum arm | $r_h$ = | 1.025 m |

Sufficient static longitudinal flight-stability must be given. As shown before, the overall aerodynamic centre of the model is determined by the formula

$$\frac{\Delta X_N}{\hat{c}} \approx \frac{a_w^\times \cdot a_h^\times \cdot A_h / A}{1 + a_w^\times \cdot a_h^\times \cdot A_h / A} \cdot \frac{r_{Nh}}{\hat{c}}$$

With $a_{ph} \approx 1.2$, $a_{pw} \leq 1.2$, and $r_h \approx r_{Nh}$ we get

Aerodynamic centre of glider $X_N/\hat{c} \approx$ 0.251 + 0.159 = 0.41

And finally the static longitudinal stability-measure for the chosen position of the c.g. corresponding to the working point $c_l = 0.9$ turns out to be

| Static stability | $\sigma$ = | $(X_N - X_{c.g.})/\hat{c}$ = 0.16 ! |
|---|---|---|

According to experience this would be quite a good stability-value for an F3J-Model.

*Following also the results of the usual conventional non-viscous calculations of the stability values are presented for comparison:*

| | |
|---|---|
| *Aerodynamic centre of the wing* | $X_{Nw}/\hat{c} = 0.25$ |
| *Zero momentum coefficient of the wing* | $c_{Mo} = -0.08$ |
| *Centre of gravity* | $\Delta X_{c.g.}/\hat{c} = -c_{Mo}/c_{Lw}(opt) = 0.08/(0.897 \cdot 0.9) = 0.0991$ |
| | ***$X_{c.g.}/\hat{c} = 0.25 + 0.099 = 0.349$*** |
| *Length of momentum arm* | ***$r_h$ = 1.004 m*** |
| *Aerodynamic centre of the glider* | $X_N/\hat{c} \approx 0.25 + 0.284 = 0.534$ |

| ***Static stability*** | ***$\sigma = (X_N - X_S)/\hat{c} = 0.19$ !*** |
|---|---|

*A comparison shows that the calculation of the static stability-measure found by consideration of viscous airstream effects as they result from the X-FOIL-profile-analysis yields a value close to that of the non-viscous stability-consideration. In principle this is due to the nearly equal shift of $X_N$ and $X_{c.g.}$ towards the leading edge of the lifting wing as a result of the viscous airstream-effects on the $c_l$- and $c_m$-derivatives as predicted by the X-Foil-program.*

***But:* *After all the experience the author has gained in designing and RC-flying of many different glider-models in over 30 years, it was never found that for a model like the one under consideration with a cambered airfoil like the "HQ/W-2.25/8.5" the c.g. should be that close at the quarter-point of the MAC for the optimum working-point ($c_l$ = 0.9) as it turns out by means of the X-Foil profile-analysis. On the contrary, in flight practice the c.g. was always found to be close to the one calculated for the optimum working-point by means of the non-viscous approach as given above. This is why the author prefers the PROFILE-program of Prof. Richard Eppler over X-FOIL, at least for calculations of flight stability. Calculation of the c.g. and stability-measures never failed when based on the PROFILE-analysis while X-FOIL always predicts a c.g. too far towards the front of the planes.***

***One major conclusion to be drawn may be that the profile-analyses as conducted by the X-FOIL-routines obviously overemphasize the viscous effects and thus predict rather strong deviations of the $\alpha$-derivatives for the lift and momentum-coefficients from the ideal non-viscous slopes, in particular for low Re-numbers. A profound revision of the parts of the program with respect of the influence of viscous effects would for sure be most appreciated by all modellers!***

g. Finally it will be of interest to which degree fast pitching-oscillations of the planned F3J-glider will be attenuated by the flight-mechanical characteristics calculated before. Having in mind what was said before about the viscous airstream-effects, the further calculations will use the non-viscous approach. In chapter 6.1.d we had shown that a disturbance of the angle of attack is described by equation 6.1.33

$$Z(t) = Z(t_o) \cdot \delta^{-\delta \cdot t} \cdot \cos(\omega \cdot t + \varphi)$$

Wherein $\omega = \omega_o^2 - \delta^2$ can be calculated according to

$$\omega_o^2 = -\frac{1}{J_y} \cdot \left( 2\pi \cdot a_w \frac{X_{c.g.} - X_{Nw}}{\hat{c}} - (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot 2\pi \cdot a_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \right) \cdot q \cdot A \cdot \hat{c}$$

$$\delta = \frac{\pi}{2J_y} \cdot a_h \cdot A_h \cdot r_h^2 \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \cdot \rho \cdot V$$

Assuming that $a_h^x \approx a_h = 0.76$, $a_w^{\times} \approx a_w = 0.897$, and $\partial\alpha_w/\partial\alpha \approx 4 \cdot a_w^{\times} / \Lambda_w = 0.2$, $\boldsymbol{\rho} = 1.25$ kg/m$^3$ we get

“Eigen”-frequency of pitch-oscillations $\omega_o = 0.53 \cdot V$ [s$^{-1}$]

Damping constant of pitch-oscillations $\delta = 0.27 \cdot V$ [s$^{-1}$]

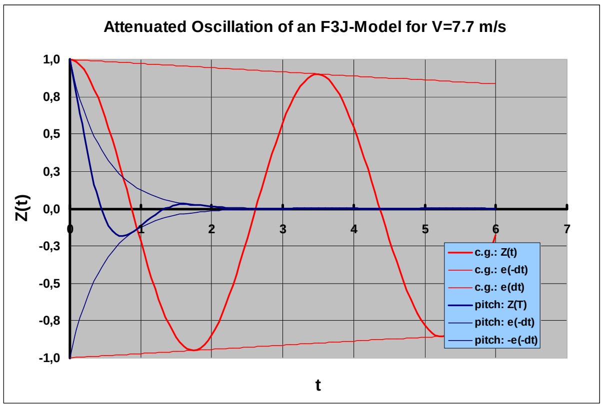
Frequency of pitch-oscillations $\omega = (\omega_o^2 - \delta^2)^{1/2} = 0.46 \cdot V$ [s$^{-1}$]

Oscillation-frequency and attenuation increase with increasing speed, thus the worst flight-state concerning damping of pitch-disturbances is given for the state related to the working-point where the flight-velocity is at its minimum

Velocity at optimum working-point $\mathbf{V} \approx 4 \cdot ((m/A)/( a_w \cdot c_a(0.9))^{1/2} = 7.7$ m/s

Here we have $\boldsymbol{\omega}_o = 4.08$ s$^{-1}$, $\boldsymbol{\delta} = 2.08$ s$^{-1}$, $\boldsymbol{\omega} = 3.54$ s$^{-1}$.

For the slow c.g.-oscillations it was shown in chapter 6.2.a that




$$\omega_{o,c.g.} \approx \sqrt{2} \cdot \frac{g \cdot \cos\vartheta}{V}$$

$$\delta_{c.g.} = \frac{g \cdot \sin\vartheta}{2 \cdot V}$$

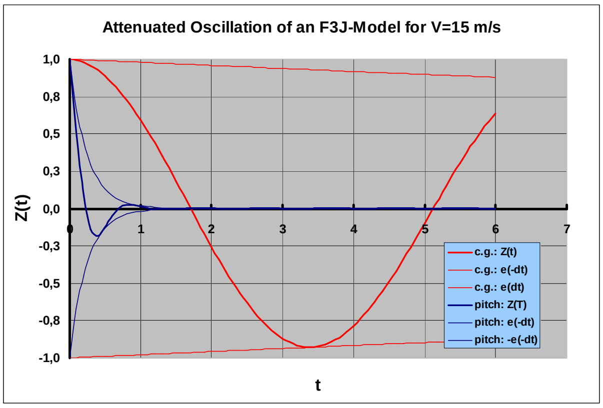
With $\boldsymbol{\vartheta} \approx 2.7°$ and V = 7.7 m/s

$\omega_{o,c.g.} = 1.80$ s$^{-1}$,

$\delta_{c.g.} = 0.030$ s$^{-1}$

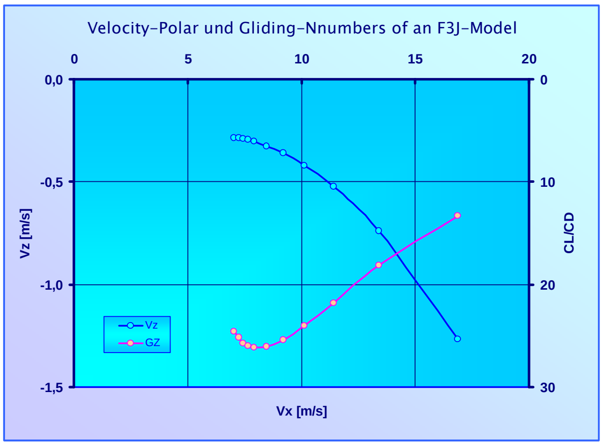
$\omega_{c.g.} = 1.80$ s$^{-1}$

The attached graphic shows, that already after one period the pitching-oscillations come to rest and the glider finds back to the stationary flight state. At low flight-velocity the slow c.g.-oscillations of the F3J-Model are only moderately damped. However, in flight practice these long c.g.-oscillations are usually not a problem; most pilots intuitively correct them with the elevator-control of the RC-transmitter.




With increasing flight-velocity also the gliding-angle decreases. As can easily be taken from the above formulae, the dampening constant $\delta_{c.g}$ decreases slightly with increasing V and $\boldsymbol{\vartheta}$, while the oscillation frequency decreases.

For example in the left hand side graphic the pitch and c.g.-oscillations are given for V= 15 m/s and $\boldsymbol{\vartheta} \approx 3.8°$.

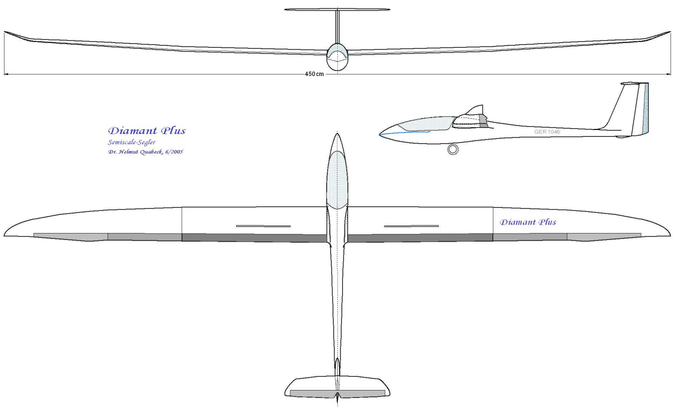



Finally the left graphic provides the theoretical performance parameters of the complete F3J-model for its expected operational flight range under inclusion of all sorts of drag related to the model.

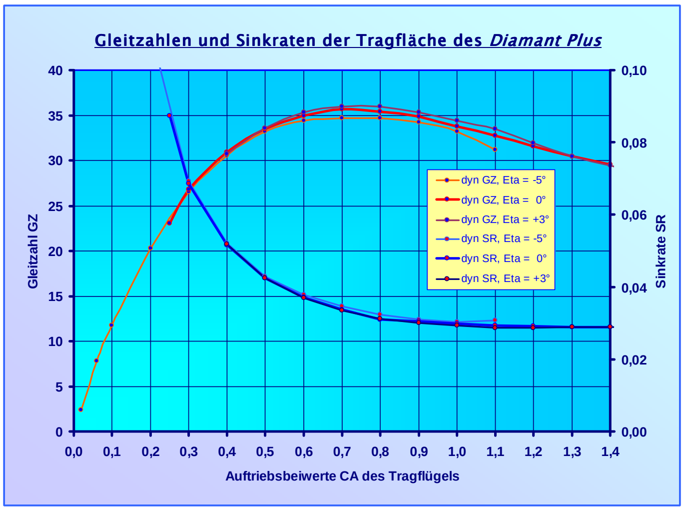
## 7.4 Example of a functional Soaring Model, "DIAMANT PLUS"

Below is given the 3 side view of the *"Diamant Plus"*, a functional glider model of the author for thermal and alpine slope soaring and model trekking, designed and built in 2005/06. The model is equipped with an electric motor, used for launching and/or as emergency return aid in the mountains.

This model is a further development of a similar model which was first launched in the early eighties.




From the original *"Diamant"* the fuselage was taken over for practical reasons und thus the momentum-arm of the model was predetermined. In our home-page www.hq-modellflug.de one can find all details about the design aspects for the new development.




The airfoil chosen for the lifting wing is the *HQ/W-3.5/13* straight from the wing root till the ends of the ailerons. From there towards the tips of the wing the sections were lofted to the *HQ/Winglet*-airfoil and twisted by about -0.7° in order to achieve good-natured stall behaviour. As can be seen in the graphic, the model is equipped with flaps and flapperons which allow to deflect the wing-sections as desired for any flight state from very slow to very high. By means of the left graphic showing gliding numbers and sinkrates for the lifting wing of the "Diamant Plus" including induced drag, the aerodynamic working point was chosen to be at $c_l = 1.2$.

The geometric and aerodynamic characteristics (calculated by the *FMFM*-program) of the model relevant for the calculation of the longitudinal flight-stability are subsequently summarized:

| | | |
|---|---|---|
| Total mass of the model | $m \approx$ | 8 kg |
| Mean chord of the wing | $\hat{c} =$ | 203.6 mm |
| Lifting area of the wing | $A =$ | 0.9162 $m^2$ |
| Load /unit-area | $m/A \approx$ | 8.8 $kg/m^2$ |
| Aspect-ratio of the wing | $\Lambda_W =$ | 22.1 |
| Lift-efficiency of the wing | $a_W =$ | 0.924 |
| Momentum-coefficient ($c_l$=1.2) | $c_{Mo} =$ | −0.135 (including a share of the fuselage) |

The centre of gravity for the chosen optimum working-point results to be at

Centre of gravity $X_{c.g.} = X_{Nw} - c_{Mo}/(a_w \cdot c_l) \cdot \hat{c}$

$= 0.0682 \text{ mm} + 0.122 \cdot 0.2036 \text{ mm} = 93.0 \text{ mm}$

Here from the minimum possible flight-velocity turns out to become

Velocity for the working point $V \approx 4 \cdot ((m/A)/(a_w \cdot c_l(1.2))^{1/2} = 11.3 \text{ m/s}$

Based on the fuselage-dimensions of the old ***"Diamant"*** a close estimate for the length of the momentum-arm between the c.g. and the approximate position of the aerodynamic centre of the elevator then is

Length of momentum-arm $r_h \approx$ 1080 mm

Using a weight-pendant for the expected elevator-mass in the position of the elevator, by means of the pendulum-method the mass-moment of inertia for the fuselage-elevator-combination related to the expected c.g. was experimentally determined to be $J_{yf} = 1.38 \text{ kg} \cdot \text{m}^2$. The mass-centre of the wing turned out to be very close to the middle of the mean chord, and related to the c.g. it was calculated to be about $J_{yw} = 4.066 \cdot 0.026 = 0.106 \text{ kg} \cdot \text{m}^2$. Thus, the total mass-moment of inertia was expected to become

| | |
|---|---|
| Mass moment of inertia | $J_y \approx 1.49 \text{ kg} \cdot \text{m}^2$ |

From the formula for the ratio of the q-derivative and the mass-moment of inertia

$$\frac{c_{m,\omega y}}{J_y} = -\frac{1}{J_y} \cdot 2 \cdot \pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2}$$

it can be taken that size $A_h$ and shape $a_h$ of the elevator are the only parameters left for adjustment of the necessary dynamic longitudinal stability-measure $c_{m,\omega y}/J_y$, since in particular the contribution of the elevator to the mass-moment of inertia growth with the length of the momentum-arm according to $J_{yh} \sim m_h \cdot r_h^2$. Because of the generally higher flight velocity of heavier models different from low-weight models like such for F3J-puroses a ratio of $c_{m,\omega y}/J_y \approx 10$ already will provide sufficient attenuation of fast pitching-oscillations after disturbances as will be shown later on. In order to achieve good lift-efficiency a double tapered elevator-shape was chosen and finally the elevator-characteristics became

Mean elevator-chord $\hat{c}_h =$ 124.6 mm

Lifting-area of elevator $A_h =$ 0.0885 m$^2$

Aspect ratio of elevator $\mathbf{\Lambda}_h =$ 5.7

Lift efficiency of elevator $a_h =$ 0.76

Pitching-attenuation $c_{m,\omega y}/J_y =$ 8.7

In order to find out which static stability will result from the chosen elevator-characteristics, next the position of the overall aerodynamic centre of the glider is to be determined by means of the formula

$$\frac{\Delta X_N}{\hat{c}} = \frac{a_w \cdot a_h \cdot A_h / A}{1 + a_w \cdot a_h \cdot A_h / A} \cdot \frac{r_h}{\hat{c}}$$

By means of the *FMFM*-program the position of the aerodynamic centre of the lifting wing was found to be at $X_{Nw}/\hat{c} = 0.335$, and with the characteristic values given before we get

Aerodynamic centre of glider $X_N/\hat{c} \approx 0.335 + 0.333 = 0.6675$

The static longitudinal stability-measure for the chosen c.g. corresponding to the working point $c_l = 1.2$ turns out to be

| Static stability | $\sigma = (\Delta X_N - \Delta X_S)/\hat{c} = 0.333 - 0.122 = 0.211$ ! |
|---|---|

This is a profound measure for the longitudinal stability of a model!

Finally the oscillatory behaviour of the glider after disturbances is of interest.

For the fast pitching oscillations an appropriate modification of equations 6.1.35 and 6.1.37 for non-viscous treatment provides

$$\omega_o \approx \sqrt{-\frac{1}{J_y}\cdot\left(2\pi\cdot a_w\frac{X_{c.g.}-X_{Nw}}{\hat{c}}-(1-\frac{4\cdot a_w}{\Lambda})\cdot 2\pi\cdot a_h\cdot\frac{A_h}{A}\cdot\frac{r_h}{\hat{c}}\right)\cdot\frac{\rho}{2}\cdot A\cdot\hat{c}\cdot V}$$

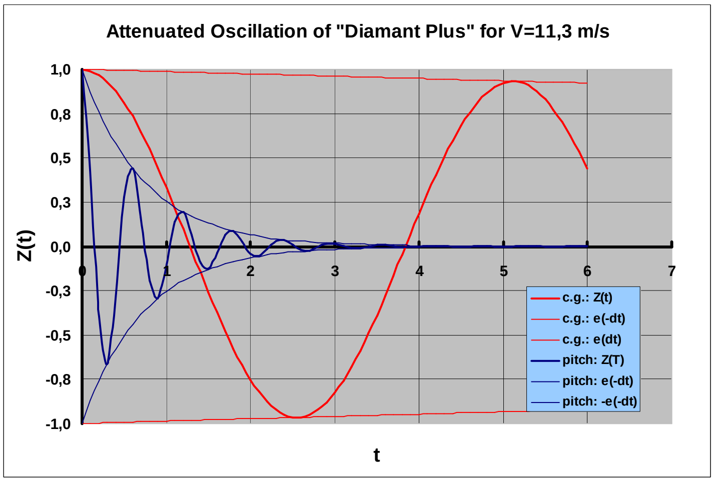
$$\delta = \frac{\pi}{2J_y}\cdot a_h\cdot A_h\cdot r_h^2\cdot(1+\frac{4\cdot a_w}{\Lambda})\cdot\rho\cdot V$$

Taking into account the foregoing characteristic values of the model we receive

| | |
|---|---|
| "Eigen"-frequency of pitch-oscillations | $\omega_o = 0.944 \cdot V\ [s^{-1}]$ |
| Damping-constant of pitch-oscillations | $\delta = 0.121 \cdot V\ [s^{-1}]$ |
| Frequency of pitch-oscillations | $\omega = (\omega_o^2 - \delta^2)^{1/2} = 0.936 \cdot V\ [s^{-1}]$ |

For the minimum velocity $V \approx 11.3$ m/s at the chosen optimum working conditions of the *"Diamant Plus"*, namely $c_l = 1.2$, we get $\omega_o = 10.67\ s^{-1}$, $\delta = 1.36\ s^{-1}$, $\omega = 10.58\ s^{-1}$

Accordingly with $\vartheta \approx 1.79°$ for the slow c.g.-oscillations at the optimum working point $c_l = 1.2$ we have $\omega_{o,c.g.} = 1.227\ s^{-1}$, $\delta_{c.g.} = 0.0136\ s^{-1}$, $\omega_{c.g.} = 1.227\ s^{-1}$.

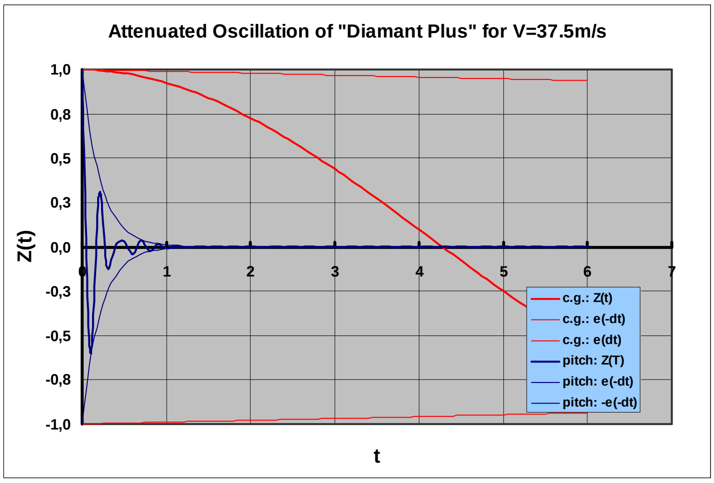



Here we have the typical behaviour of a model with a rather high mass-moment of inertia. While for the previous lower-weight F3J-model the fast pitching-oscillations already came to rest after about one cycle, here it takes about 3 cycles.

As to be expected, the attenuation of the slow c.g.-oscillations is of the same order of magnitude.

Although the attenuation of the pitch oscillation appears to be lower boarder, extensive flight practice with the *"Diamant Plus"* over a full season on flat fields and in the mountains have proven that this in no way is insufficient.

Without further explanations in the last graphic the fast pitch and slow c.g.-oscillations and their attenuation after disturbances are given as they will appear at higher flight-velocity:




As we see, the fast pitching-oscillations very soon come to rest, whilst the c.g.-oscillations take a long time and can easily be balanced out by RC-control.

*The **major conclusions** which can be drawn from this example for glider-models with higher mass-load are*

- *The mass-moment of inertia should be kept as low as possible in order to achieve the best possible dynamic longitudinal stability, in particular this hold true for acrobatic-gliders,*
- *As pointed out repeatedly, the weight of the model tail should be kept as low as possible, because the tail has the largest distance of all parts to the c.g. and thus contributes most to the mass-moment of inertia,*
- *For scale-gliders the size and the shape of the elevators are given by the original. It often happens, that these elevator-proportions are not sufficient for a stable flight-behaviour since they do not provide the required contribution at model-scale. In those cases it may not disturb the scale impression when the span-width of the model is increased by 10 to 15 %.*
- *At functional models with higher load the dynamic stability can be influenced by the length of the momentum-arm as well as by shape and size of the elevator. While designing such a model it has always to be kept in mind that attenuation by the elevator is counterbalanced by its mass moment of inertia!*

## 8. Final Recommendations

Whilst the longitudinal stability behaviour oft the above ***F3J***- and "***Diamant Plus***"-examples was determined, it was already indicated how this could best be performed. Concluding, recommendations will be given for a more universal proceeding at the design of a plane with required longitudinal stability behaviour.

### ➢ Design of a functional plane

1. When designing a new functional model as for F3-classes, acrobatic flying, or free just-for-fun-flying, the first step should be to determine the dimensions and shape for the lifting wing. Thereat usually major attention should be paid to a good lift-efficiency of the wing, expressed by the shapefactor $a_W$. E.g. this efficiency can exactly be calculated by means of the ***FMFM***-program of the author, for a quasi-elliptical wing-shape it is approximately given by consideration of the aspect ratio: $a_W \approx \Lambda_W/(2+ (\Lambda_W^2 + 4))^{1/2}$.

2. In a second step the quasi-stationary $c_l$-$c_d$-polar corresponding to the expected wing-load m/A of the plane should be determined for the airfoil of the lifting wing. From these polars the

corresponding quasi-stationary sinkrates and $c_L/c_D$-ratios for the lifting wing can be developed as functions of $c_l$, where $c_D$ should include the airfoil- and the induced drag of the wing. As shown for the examples above, from these curves the optimum $c_l$-working-point can be determined either for best gliding-angle or minimum sinkrate.

**3.** In a third step the position for the centre of gravity c.g. should be fixed according to equation 7.3.6 in section 7.3.c. Here the problem appears that the position of the aerodynamic centre of the wing $X_{Nw}$ is affected by viscous airstream influences on lift and momentum of the chosen airfoils, in particular at lower Re-numbers. As discussed earlier, if these effects are determined by *X-FOIL*-analyses of the wing-sections the calculated position of $X_{Nw}$ does not well coincide with practical experience, while the *PROFILE*-program supplies reliable results which are close to the quarter-point of the MAC. For normal planes with sufficient accuracy the c.g. can be chosen according to $X_{c.g.}/\hat{c} = 0.25 - c_{Mo}/c_{Lw}(opt)$. This c.g. choice also leaves room for flight states with non-zero lift at the elevator and in particular for the up and down deflection of flaps.

**4.** In a fourth step, next the value for the static stability measure $\sigma = (X_N - X_{c.g.})/\hat{c}$ needs to be chosen. This measure is often also given in percentages of the MAC. According to experience lower weight models will already fly quite stable with 10 % stability, however, models with higher weight should better have 15 - 20 % static stability or even more. With chosen $\sigma$ and $X_{c.g.}$ the necessary position of the overall aerodynamic centre $X_N$ for the required static stability follows. Then, by means of equation 4.17 and under the assumption that $a_h^x \approx a_h$, $a_w^x \approx a_w$, and $r_{Nh} \approx r_h$ an idea for the size $A_h$ of the elevator and its momentum arm $r_h$ can be developed as shown in the examples. For aerodynamic reasons, namely in order to keep the drag of the elevator as low as possible, it may be advisable to choose the elevator area $A_h$ as small as the aerodynamic characteristics of the elevator-airfoil allow and to compensate this with a longer momentum arm. E.g. for larger F3J-models $A_h/A \approx 0.09$ would be sufficient in order to achieve appropriate aerodynamic elevator performance with the airfoil *HQ/W-0/9*.

**5.** In the fifths step at least a rough idea should be developed for the mass-moments of inertia $J_y$ related to the c.g., in particular also for that of the tail part which contributes most to the overall value according to $J_{y;tail} \sim m_{tail} \cdot r_{tail}^2$. It cannot be repeated often enough, the weight of the tail and the rear part of the fuselage should be as low as possible in order to achieve good dynamic longitudinal stability.

**6.** Once the c.g. and the overall aerodynamic centre $X_N$ are defined by the choice of the elevator dimensions $a_h \cdot A_h$ and its distance $r_h$ from the c.g., then with the estimated mass-moment of inertia $J_y$ the attenuation of disturbances of the angle of attack, gliding angle and velocity are also determined. A closer look to the formulas for the attenuated fast pitch oscillations appearing after disturbance of the angle of attack tells us that both the pitching-moment $c_{m,\omega y} \sim A_h \cdot r_h^2$ and the major contribution to the mass-moment of inertia $J_{yh} \approx m_h \cdot r_h^2$ depend on the second power of $r_h$. Consequently the major contributions to the attenuation coefficient of the fast pitching-oscillations result from size and mass of the elevator and changes proportionally to the flying velocity according to

$$\delta \sim A_h / m_h \cdot V$$

This again demonstrates how important the weight of the elevator (and that of fin and rear fuselage as well) is for fast damping of pitch-disturbances. (*Here from the author's preference for light-weight V-tails originates.*) Accordingly the frequency of the fast pitching-oscillations is mainly determined by

$$\omega \sim \sqrt{A_h / m_h} \cdot \sqrt{1/r_h} \cdot V$$

The frequency of the fast pitching oscillations increases with the square-root of the elevator size and with the velocity while it decreases with the square-roots of the tail-weight and the elevator-momentum-arm. Thus, as already required for other reasons before, a smaller elevator and a longer momentum arm as required for other reasons before will help to keep the oscillation-frequency low. Although a higher tail-weight would reduce the oscillation-frequency, for reasons mentioned before, low tail weight is to be preferred.

## Design of a Scale-Model

**1.** When designing a scale-model, the first step should be to determine the dimensions and shape for the lifting wing and the elevator from the corresponding data of the original. There from the lift-efficiency-factors $a_w$ and $a_h$ are to be determined. E.g. this efficiency can exactly be calculated by means of the ***FMFM***-program of the author, for a quasi-elliptical wing-shape (which is applied for almost all modern gliders) they can approximately be calculated by means of the aspect ratios: $a_w \approx \Lambda_w/(2+ (\Lambda_w^2 + 4))^{1/2}$ and $a_h \approx \Lambda_h/(2+ (\Lambda_h^2 + 4))^{1/2}$.

**2.** In a second step like for the functional planes the quasi-stationary $c_l$-$c_d$-polar corresponding to the expected wing-load m/A should be determined for the airfoil of the lifting wing. From these polars the corresponding quasi-stationary sinkrates and $c_L/c_D$-ratios for the lifting wing can be developed as functions of $c_l$, where $c_D$ should include the airfoil- and the induced drag of the wing. As shown for the examples above, from these curves the optimum $c_l$-working-point can be determined either for best gliding-angle or minimum sinkrate of the scale-plane.

**3.** In a third step the position for the centre of gravity $X_{c.g.}$ should be determined according to equation 7.3.6 in section 7.3.c. As discussed earlier for the functional models, here the problem may appear that the position of the aerodynamic centre of the wing $X_{Nw}$ is affected by viscous airstream influences on lift and momentum of the chosen airfoils, in particular at lower Re-numbers. But in general for normal planes with sufficient accuracy the c.g. can be chosen according to $X_{c.g.}/\hat{c} = 0.25 - c_{Mo}/c_{Lw}(opt)$. Again as before this c.g. choice also leaves room for flight states with non-zero lift at the elevator and in particular for the up and down deflection of flaps.

**4.** As soon as the position of the c.g. is determined, the length of the momentum-arm $r_h$ (the distance of the aerodynamic centre of the elevator from the c.g.) can be determined, and based on the geometric data of the model and the wing and elevator efficiencies, $a_w$ and $a_h$, the position of the overall aerodynamic centre of the scale-model can be found by means of equation 4.1.17, and finally the static stability measure $\sigma$. If the static stability of a larger scale-glider should turn out to be $\sigma < 0.15$ then the stall behaviour of the model at slow soaring may become critical. In such cases an enlargement of the elevator-span should be considered, since this would not harm the scale impression and would be the easiest way to improve the static stability. Unfortunately the static stability of Old-timer-glider often also suffers from too short distances of the elevator from the c.g., in theses cases it may be advisable to also lengthen a bit the rear fuselage-part.

**5.** In the further step at least a rough idea should be developed for the mass-moments of inertia $J_y$ related to the c.g., in particular also for that of the tail parts which contribute most to the overall value according to $J_{y;tail} \sim m_{tail} \cdot r_h^2$. The weight of the tail and the rear part of the fuselage should be as low as possible in order to achieve good dynamic longitudinal stability. In particular larger scale models often carry a lot of unnecessary weight along in their tail parts. E.g. since large scale-gliders offer much space in the fin, often heavy, strong servos are mounted therein.

**6.** Once the c.g. is defined, then with the estimated mass moment of inertia $J_y$ the attenuation of disturbances of the angle of attack, gliding angle and velocity can be derived and it can be seen which dynamic stability behaviour the scale model may develop for certain flight-conditions. However, there is no further parameter to be found by these considerations which could be changed to influence the dynamic flight behaviour.