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On the Longitudinal Flight Stability of Gliders

By Helmut Quabeck, ©, updated 3/2023

Abbreviations

(***a₁, a₂, a₃***) system of coordinates fixed to the flight-wind

(***f₁, f₂, f₃***) system of coordinates fixed to the airplane

c = chord length

$\hat{c}$ = mean aerodynamic chord length (MAC)

c_l = lift-coefficient of the profile

c_L = lift-coefficient

c_{Lw} = lift-coefficient of the wing

c_{Lh} = lift-coefficient of the elevator

c_d = drag-coefficient of the airfoil

c_{Dw} = drag coefficient of the wing

c_{Di} = induced drag-coefficient of the wing

c_{mo} = momentum-coefficient of the airfoil related to its aerodynamic centre

c_M = momentum-coefficient of plane

c_{Mo} = momentum-coefficient of the lifting wing related to its aerodynamic centre

r_h = distance of the aerodynamic centre of the elevator from the c.g.

X,Y,Z = inertia-forces in the system fixed to the flight-wind

R = Vektor der gesamten Luftkraft

A = wing-area

A_h = elevator-area

L = lift

L_w = lift of the wing

L_h = lift of the elevator

D = drag

D_w = drag of the lifting wing

D_i = induced drag

M = pitching-moment of the glider

M_o = pitching-moment of the wing related to the aerodynamic centre

X_N = position of the aerodynamic centre of the glider

X_{Nw} = position of the aerodynamic centre of the lifting wing

Xc.g. = position of the centre of gravity

V = velocity of the glider

u = peed of sound

Ma = V/u, Mach-Number

α = angle of attack

α_o = angle of attack at zero-lift

α_g = geometric angle of attack, $\alpha_g = \alpha + \alpha_o$

α_w = downwash-angle

ε = difference of the angles of incidence of elevator and wing

$\acute{\alpha}$ = $d\alpha/dt$ rotational speed of the angle of attack

ϑ = angle of inclination, gliding-angle

ε = difference of angles of incidence

Λ_w = aspect-ratio of the wing

a_w = lift efficiency-factor of the lifting wing

Λ_h = aspect-ratio of the elevator (shape influence)

a_h = lift efficiency-factor of the elevator (shape-influence)

q = ω_y = rotational speed around the lateral y-axis through the c.g.

$\bar{q}$ = aerodynamic pressure, $\bar{q} = \rho/2 \cdot V^2$

ρ = airdensity, ≈ 1.25 kg/m^3 at sea-level

σ = measure of the static stability

m = body-mass

m_i = mass of model part i (e.g. wing, fuselage, elevator,…)

r_i = distance of the mass-centre of model-part i from the c.g.

J_y = mass-moment of inertia related to y-axis (c.g.)

J_{yi} = mass-moment of inertia of model-part i related to y-axis (c.g.)

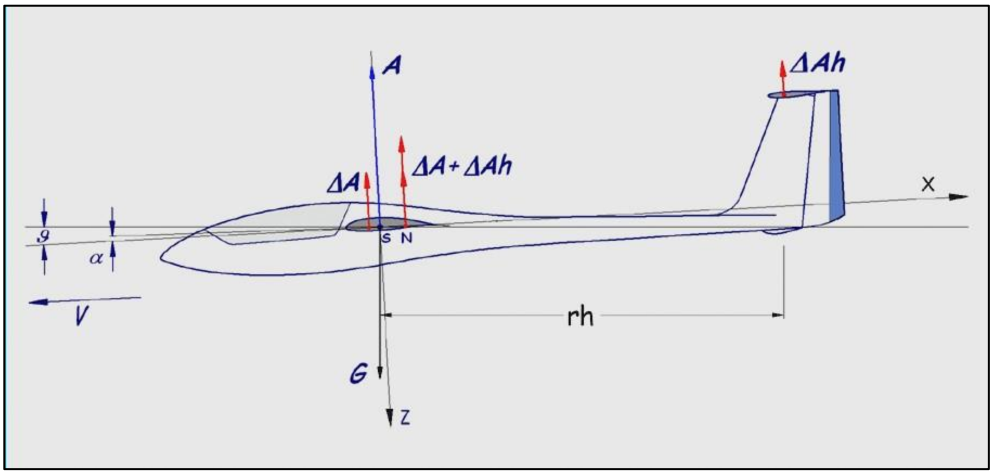
F(n) = characteristic equation with n solutions

$\lambda_{1;2}$ = solutions of F(4) for α-disturbances-

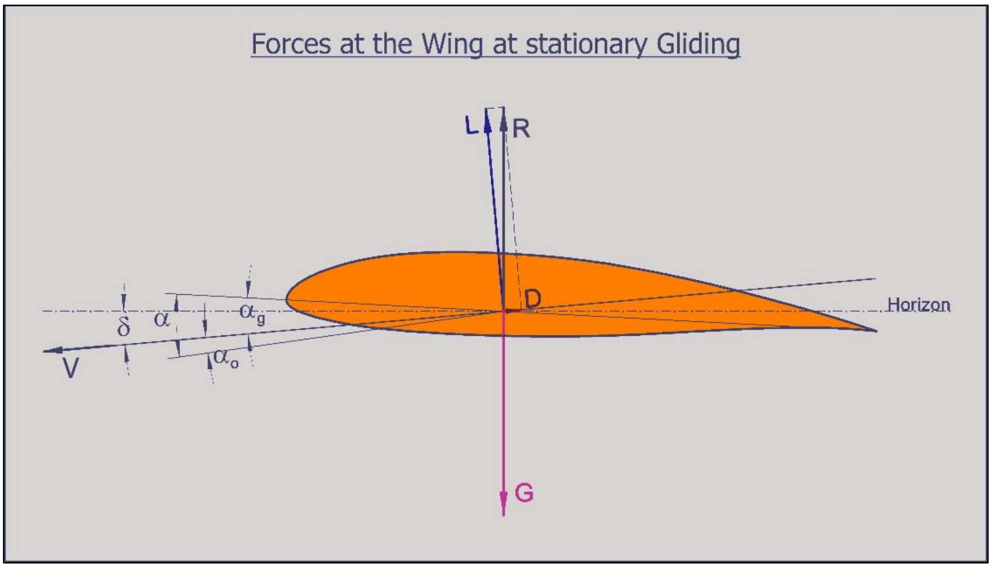
$\lambda_{3;4}$ = solutions of F(4) for ϑ-V-disturbances

$\partial y(x_1,x_2,\ldots)/\partial x_n$ partielle Ableitung von y nach x_n

Simplified Graphic Overview of Forces on a Glider (A=L, W=D)



Detailed Graphic of Air Forces



1. Forces at the Glider at longitudinal Motion

Usually the forces acting on an airplane are disassembled such that they match the coordinate system fixed to the flight wind (***a₁, a₂, a₃***), where ***a₁*** corresponds with the flight direction ***V***. The resulting of all air forces on an airplane is designed by the vector ***R***. Its component in the flow direction - ***a₁*** is usually described as drag ***D***. The component of ***R***, vertical to the direction of the air flow - ***a₁***, is described as lift ***L***, pointing in the direction of - ***a₃***. The 3rd unit coordinate then is ***a₂*** = ***a₃*** x ***a₁*** (direction of wingspan).

In the coordinate system fixed to the flight direction the equations of the forces acting on a glider, namely the forces due to mass inertia, aerodynamic lift and drag, and the total weight G (motor of gliding), are

a₁ – direction:

$$X = m \cdot \dot{V}$$

$$X = m \cdot g \cdot \sin\vartheta - D \qquad (2.1)$$

a₃ – direction:

$$Z = m \cdot V \cdot \dot{\vartheta}$$

$$Z = m \cdot g \cdot cos\,\vartheta - L \qquad (2.2)$$

2. Pitching-Moment of the Glider

Different to the air forces the air power moments acting on a glider in flight are preferably disassembled according to the coordinate system fixed to the airplane (***f1, f2, f3***). Of particular interest here is the pitching moment M around the transverse axis (***f2***) of the airplane:

The pitching-moment M caused by air forces at the glider in general depends on

- the angle of attack α,
- the Mach number, which mostly can be neglected at the lower speed of model-gliders,
- the rotational speed of the angle of attack $\acute{\alpha} = d\alpha/dt$,
- and the rotational speed around the transverse axis, ω_y

$$M = M(\alpha, Ma, \acute{\alpha}, \omega_y) \qquad (3.1)$$

$M = M(\alpha, \acute{\alpha}, \omega_y)$ at low speed, < 80 m/s

Using a non-dimensional momentum coefficient c_m, according to aerodynamic theory we get

$$M = c_m(\alpha, Ma, \acute{\alpha}, \omega_y) \cdot q \cdot A \cdot \hat{c} \qquad (3.2)$$

First order Taylor derivation of the momentum-coefficient c_m provides

$$c_m = c_m(\alpha,0,0) + \frac{\dot{\alpha} \cdot \hat{c}}{V} \cdot c_{m,\dot{\alpha}} + \frac{\omega_y \cdot \hat{c}}{V} \cdot c_{m,\omega_y} \qquad (3.3)$$

Therein the derivatives $c_{m\acute{\alpha}} = \partial c_m / \partial\acute{\alpha}$ and $c_{m\omega y} = \partial c_m / \partial q$ are dependent on α.

The first term in this equation represents the pitching moment coefficient of the airplane for the centre of gravity $X_{c.g.}$ at fixed rudders.

The second term represents the coefficient of a damping-moment which is proportional to the angular velocity $\acute{\alpha}$ of the angle of attack α. Essentially it results from the fact that the angle $\alpha_w(t)$ of the downwash, resulting from the lifting wing at the elevator in case of $\acute{\alpha} \neq 0$, does not correspond to the angle of attack $\alpha(t)$ of the wing, but to the angle $\alpha(t + \Delta t)$. $\Delta t \approx r_h/V$, and r_h is about the distance of the aerodynamic centre of the elevator from the centre of gravity, Xc.g. E.g. for $\acute{\alpha} > 0$ the downwash angle α_w gets smaller and the resulting angle of attack at an elevator within the downwash of the wing becomes $\alpha_h = \alpha\,(t) - \alpha_w(t + \Delta t)$. Thus the lift at the elevator in this case becomes larger then for the stationary case with $\acute{\alpha} = 0$, a negative pitching-moment will result, and in general we have $c_{m\acute{\alpha}} < 0$.

The third term represents the coefficient of a dampening-moment which is proportional to the rotational speed $q = \omega_y$ around the lateral axis through the Xc.g. It results from the fact that the angle of attack and thus the lift at the elevator in case of $\omega_y > 0$ is increased by the angle $\omega_y \cdot \hat{c}/V$ as compared to the stationary state with $\omega_y = 0$. Again a negative pitching-moment results with $c_{m\omega y} < 0$.

These aerodynamic pitching-moments are counteracted by the mass-inertia of the glider-parts expressed by their moments of inertia J_y around the y-axis of the glider**.** May m_i be the mass of a glider part and r_i its distance from the c.g., then the mass-moment of inertia of the glider can roughly be assessed to be

$$J_y = \sum_i m_i \cdot r_i^2 \tag{3.4}$$

Thus for the pitching moment generally we have

$$M = J_y \cdot (\frac{\partial^2 \vartheta}{\partial t^2} + \frac{\partial^2 \alpha}{\partial t^2}) = J_y \cdot (\frac{\partial q}{\partial t} + \frac{\partial \dot{\alpha}}{\partial t}) \tag{3.5}$$

3. Aerodynamic Centre of the Glider, X_N

The aerodynamic centre of the glider is defined as the point X_N on its longitudinal axis where the pitching moment is constant with respect to the angle of attack. Thus, in case of a change of the angle of attack the resulting lift L will act through X_N.

Equally the aerodynamic centre of the lifting wing is defined as the point X_{Nw} around which the pitching moment of the wing, in generally dependent on the wing-shape and the properties of the chosen airfoils, is constant with respect to the angle of attack. Since the lift- and momentum-characteristics, $c_l(\alpha)$ and $c_m(\alpha)$, of most airfoils show a slightly non-linear dependence on the angle of attack for low Reynold-numbers, $Re < 1 \cdot 10^6$, the aerodynamic centre of wings can only be considered to be stable for small α-ranges. This has to be taken into account at the design of a model-plane in order to achieve proper static and dynamic flight stability under all flight conditions.

The displacement of the overall aerodynamic centre of the plane X_N vs. the aerodynamic centre of the wing X_{Nw} may be denoted by $\Delta X_N = X_N - X_{Nw}$. The momentum-balance of the plane is then given by

$$\Delta X_N \cdot L = r_{Nh} \cdot L_h \tag{4.1}$$

Herein L_h is the contribution of the elevator to the overall lift of the plane, and r_{Nh} is the distance of the aerodynamic centres of lifting wing and elevator. According to aerodynamic theories we get

$$L_h = c_{lh} \cdot A_h \cdot q_h \ , \qquad \text{where } q_h = \rho/2 \cdot V^2 \tag{4.2}$$

c_{lh} is the lift coefficient of the elevator related to the area A_h of the elevator and the aerodynamic pressure q_h at the location of the elevator, whereat ρ is the density of the air. For practical reasons it is preferred to relate the lift coefficient of the elevator to the wing area A and its aerodynamic pressure q :

$$L_h = c_{Lh} \cdot A \cdot q \tag{4.3}$$

By comparison results

$$c_{Lh} = c_{lh} \cdot A_h / A \cdot q_h / q \tag{4.4}$$

If the airflow on the elevator would not be influenced by the wake from the wing, we would get the derivative

$$c_{Lh,\alpha} = \frac{\partial c_{Lh}}{\partial \alpha} \cdot \alpha = \frac{\partial c_{lh}}{\partial \alpha} \cdot \alpha \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \tag{4.5}$$

However, when downwash **w** in the wake of the wing affects the elevator (e.g. that of a cross tail, T-tails are less affected), the angle of attack at the elevator may be reduced by the downwash angle $\alpha_w = w/V$. The difference of the angles of incidence between wing and elevator may be denoted by ε , then the angle of attack at the elevator is given by

$$\alpha_h = \alpha + \alpha_w + \varepsilon \tag{4.6}$$

Taking this angle of attack into account, the coefficient of the elevator lift results to be

$$c_{Lh} = c_{lh,\alpha} \cdot (\alpha + \alpha_w + \varepsilon) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \tag{4.7}$$

Thus, at longitudinal motion of the plane in case of downwash the lift coefficient of an affected elevator in dependence on α is given by the derivative $c_{Lh,\alpha}$:

$$c_{Lh,\alpha} = c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.8)$$

Then the overall lift-coefficient is

$$c_L = c_{Lw} + c_{Lh} \qquad (4.9)$$

Thus, in the lift-deviation $c_{L,\alpha}$ of the whole plane in case of fixed controls with downwash results to be

$$c_{L,\alpha} = c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q} \qquad (4.10)$$

Using formulas 4.8 and 4.10 in formula 4.1, we will receive

$$\Delta X_N = \frac{c_{Lh,\alpha}}{c_{L,\alpha}} \cdot r_{Nh} \qquad (4.11)$$

Related to the mean aerodynamic chord of the wing we in the case of downwash we finally get

$$\frac{\Delta X_N}{\hat{c}} = \frac{c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}}{c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 + \frac{d\alpha_w}{d\alpha}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}} \cdot \frac{r_{Nh}}{\hat{c}} \qquad (4.12)$$

In a larger distance behind the wing the free vortices of the wing induce a downwash angle of about $\alpha_{w\infty} = -2\, c_{Lw} / (\pi \cdot \Lambda_w)$. There from results $\partial \alpha_{w\infty} / \partial \alpha = -2\, c_{Lw,\alpha} / (\pi \cdot \Lambda_w)$, and in case that the elevator is being affected by the downwash, for the aerodynamic centre of the plane we have

$$\frac{\Delta X_N}{\hat{c}} = \frac{c_{lh,\alpha} \cdot (1 - \frac{2c_{Lw,\alpha}}{\pi \cdot \Lambda_w}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}}{c_{Lw,\alpha} + c_{lh,\alpha} \cdot (1 - \frac{2c_{Lw,\alpha}}{\pi \cdot \Lambda_w}) \cdot \frac{A_h}{A} \cdot \frac{q_h}{q}} \cdot \frac{r_{Nh}}{\hat{c}} \qquad (4.13)$$

If the elevator is not affected by downwash, then $\partial \alpha_{w\infty} / \partial \alpha$ may be neglected. This formula is still rather complex and for most modellers difficult to solve. A way out of this dilemma is found for practical cases when considering how the derivatives $c_{Lw,\alpha}$ and $c_{lh,\alpha}$ depend on the lifting characteristics of the chosen airfoils and on the shapes of wing and elevator.

The slope of the lift-coefficients of a lifting wing, $c_{Lw,\alpha}$, and the elevator, $c_{Lw,\alpha}$, are closely related to the slope of the lift-coefficient of the applied airfoils, namely $c_{lh,\alpha}$, by an efficiency factor denoted as a_w

$$c_{Lw,\alpha} = a_w \cdot c_{l,\alpha} \qquad (4.14)$$

a_W takes into account the influence of the wing-shape and the airfoil layout on the formation of the free vortices on the wing-surfaces. According to the limited wing-span and of the wing-shape the ideal lifting-efficiency of the airfoils is reduced. In an ideal non-viscous environment, the slope of an ideal airfoil would be $c_{l,\alpha} = 2\pi$. However, in a viscous airflow for lower Re-numbers non-linear deviations from this ideal slope may be experienced, in the non-critical range of the angles of attack an increase up to 5% can be experienced. This can be taken into account by the profile-efficiency. Thus, we get

$c_{Lw,\alpha} = a_w \cdot 2 \cdot \pi$ (4.15)

$c_{Lh,\alpha} = a_h \cdot 2 \cdot \pi$ (4.16)

After a few rearrangements, for larger aspect ratios and elevator positions outside of the wake from the wing (e.g. lower cross-tail, T-tail) formula 4.13 can be simplified into a practically easier form:

$$\frac{\Delta X_N}{\hat{c}} = \frac{\frac{a_h}{\cdot a_w} \cdot A_h/A}{1+\frac{a_h}{\cdot a_w} \cdot A_h/A} \cdot \frac{r_{Nh}}{\hat{c}} \quad (4.17)$$

therein a_w and a_h denote the total lifting-efficiencies of wing and elevator.

***Remark**: The lifting efficiency factors a_w and a_h can easily and exactly be determined by means of the **"FMFM"**-program of the author.*

For many practical cases, if the aspect-ratio $\Lambda \geq 5$ and the sweep of the wing < 8°, in accordance with the expanded lifting line theory the efficiency-factors can be approximated by

$$a = \boldsymbol{k} \cdot \Lambda / (2 + \sqrt{\Lambda^2 + 4}) \quad (4.18)$$

Wherein $\boldsymbol{k}$ takes into account the lifting efficiencies of the wing-form and the of the distribution of the wing-sections. For nearly elliptical wing-shape and lower viscosity effects of the profiles $\boldsymbol{k} \approx 0$.

4. Static longitudinal stability, σ

One of the most important flight mechanical characteristics of a glider is the capability, to restore the balance of the original stationary longitudinal flight state after disturbance of the angle of attack by $\Delta\alpha$ without using controls. A disturbance of the angle of attack causes an increase or decrease of the lift by ΔL of the plane, if thereby a momentum ΔM is caused that forces the plane to rotate back to the original state, the plane is featured statically stable. Thus, a glider behaves statically stable if generally holds true that

$$dM = -\sigma \cdot dL \quad (5.1)$$

σ is a non-dimensional positive constant factor which is considered as a stability-measure for the stationary longitudinal flight of a glider. The larger it is, the larger the back-leading momentum will be. For **σ** = 0 the stability-behaviour of the plane will be indifferent and it will no more be controllable, for **σ** < 0 the longitudinal flight of the plane will become instable.

***Note:** As will be shown later, besides **σ** also the mass-moments of inertia Jy of the glider parts are to be taken into account to completely determine the time-dependent motion of a glider back to flight-balance after disturbance.*

Using non-dimensional aerodynamic coefficients, we get

$$\sigma = -dc_M / dc_L \quad (5.2)$$

Based on the explanations in chapter 2 the pitching moment around the c.g. is given by

$$M = -(X_N - X_{c.g.}) \cdot L + M_{ow} - r_h \cdot A_h + M_{oh} \quad (5.3)$$

Therein M_{ow} is the pitching-moment for the wing at the aerodynamic centre, M_{oh} is that of the elevator, and r_h is the distance of the aerodynamic centre of the elevator from the c.g. Moments according to the vertical position of the forces can mostly be neglected for gliders. For the change of the pitching-moment around the c.g. by change of the lift here from results

$$dc_M / dc_L = -(X_N - X_{c.g.}) / \hat{c} \qquad (5.4)$$

Implementing (3.4) in (3.2) yields

$$\boxed{\sigma \ = \ (X_N - X_{c.g.}) / \hat{c}} \qquad (5.5)$$

Therewith we have a very useful, quantitative measure for the static stability of the glider, namely the distance of the c.g. from the aerodynamic centre of the airplane related to the mean aerodynamic chord of the lifting-wing. Because of the requirement $\sigma > 0$, the c.g. must be positioned in front of X_N in order to achieve longitudinal static flight stability.

As will be discussed in more detail later on, usually the position of the centre of gravity should be chosen such that the lift coefficient c_L at slow stationary gliding is either adapted to optimum gliding or to the minimum sinkrate, or somewhere in between.. Once the c.g. is determined by evaluation of the profile- and wing-characteristics, by means of the theoretical considerations in chapter 4 the geometrical parameters of the glider can be chosen such that the required size of σ will be achieved. One problem here may be, how it can be found out which the appropriate size of σ is. The most adequate way is to determine the static stability of one or more similar representative models which are considered to have good stability-behaviour.

5. Free Oscillations of a Glider with Fixed Controls

The static stability-measure σ in principle just provides an answer to the question whether or not a glider will behave stable on a stationary linear flight path. However, it does not inform how fast the glider will restore the original stationary balance after any disturbance. In case of static stability we can expect that the glider carries out oscillations. In the most general case, a glider may conduct combined α- and ϑ-oscillations as well as oscillations of the c.g along the gliding path. As will be outlined later on, in most practical cases the α-oscillations are much faster than the c.g.-oscillations and by means of appropriate choice of the glider-design-parameters it can be achieved, that these oscillations are to such a degree that the glider returns to balance in a very short time. c.g.-oscillations take longer and cannot so well be attenuated, however, however, in practice they can easily be balanced out by proper RC-controlling of the pilot.

In order to determine the behaviour of a glider after disturbance of the angle of attack. and/or the gliding angle this movements may be considered as small disturbances ΔV, $\Delta\alpha$ und $\Delta\vartheta$ of a stationary linear flight path. Then the equations for the forces at the glider, given in chapter 2, may be developed into *Taylor*-series whereby higher power elements are neglected:

$$\boxed{m \cdot \dot{V} = X_V \cdot \Delta V + X_\vartheta \cdot \Delta\vartheta + X_\alpha \cdot \Delta\alpha}$$

$$X_V = -\partial D / \partial V$$

$$X_\alpha = -\partial D / \partial \alpha \qquad (6.1)$$

$$X_\vartheta = -m \cdot g \cdot \cos\vartheta$$

and

$$\boxed{m \cdot V \cdot \dot{\vartheta} = Z_V \cdot \Delta V + Z_\vartheta \cdot \Delta\vartheta + Z_\alpha \cdot \Delta\alpha}$$

$$Z_V = \partial L / \partial V$$

$$Z_\vartheta = m \cdot g \cdot \sin\vartheta \qquad (6.2)$$

$$Z_\alpha = \partial L / \partial \alpha$$

Equally the momentum equation of chapter 3 is developed to

$$J_y \cdot (\frac{d^2\Delta\vartheta}{dt^2} + \frac{d^2\Delta\alpha}{dt^2}) = M_V \cdot \Delta V + M_\alpha \cdot \Delta\alpha + M_{\dot\vartheta} \cdot \dot\vartheta + \overline{M}_{\dot\alpha} \cdot \dot\alpha$$

$$M_V = \partial M(\alpha,0,0) / \partial V \cdot q \cdot A \cdot \bar{c}$$

$$M_\alpha = \partial M(\alpha,0,0) / \partial\alpha \cdot q \cdot A \cdot \bar{c} \qquad (6.3)$$

$$M_{\dot\vartheta} = \bar{c}/V \cdot c_{m,\omega} \cdot q \cdot A \cdot \bar{c}$$

$$\overline{M}_{\dot\alpha} = \bar{c}/V \cdot (c_{m,\dot\alpha} + c_{m,\omega}) \cdot q \cdot A \cdot \bar{c}$$

Rearrangement of the force equations 6.1 and 6.2 and of the momentum equation 6.3 provides

$$\begin{array}{llll} (m \cdot \frac{d}{dt} - X_V)\Delta V & - X_\vartheta \cdot \Delta\vartheta & - X_\alpha \cdot \Delta\alpha & = 0 \\ - Z_V \cdot \Delta V & + (m \cdot V \cdot \frac{d}{dt} - Z_\vartheta)\Delta\vartheta & - Z_\alpha \cdot \Delta\alpha & = 0 \\ - M_V \cdot \Delta V & + (J_y \cdot \frac{d^2}{dt^2} - M_{\dot\vartheta})\Delta\vartheta & + (J_y \cdot \frac{d^2}{dt^2} - \overline{M}_{\dot\alpha} \cdot \frac{d}{dt} - M_\alpha)\Delta\alpha & = \mathbf{0} \end{array} \qquad (6.4)$$

By means of an exponential description of the disturbances according to

$$\Delta V = \Delta V_o \cdot e^{\lambda t}, \quad \Delta\boldsymbol{\alpha} = \Delta\boldsymbol{\alpha}_o \cdot e^{\lambda t}, \quad \Delta\boldsymbol{\vartheta} = \Delta\boldsymbol{\vartheta}_o \cdot e^{\lambda t}$$

the characteristic equation of the system $F_4(\lambda)$ becomes:

$$m^2 \cdot V \cdot J_y \cdot F_4(\lambda) \equiv \begin{vmatrix} m \cdot \lambda - X_V & - X_\vartheta & - X_\alpha \\ - Z_V & m \cdot V\lambda - Z_\vartheta & - Z_\alpha \\ - M_V & Jy \cdot \lambda^2 - M_{\dot\vartheta} \cdot \lambda & J_y \cdot \lambda^2 - \overline{M}_{\dot\alpha} \cdot \lambda - M_\alpha \end{vmatrix} \qquad (6.5)$$

In flight-mechanical theories this equation is usually written in the subsequent form

$$\lambda^4 + B \cdot \lambda^3 + C \cdot \lambda^2 + D \cdot \lambda + E = 0 \qquad (6.6)$$

According to the stability criteria of *Hurwitz* for an oscillating system like the one considered

$$E > 0! \qquad (6.7)$$

is a necessary requirement for the longitudinal stability of the glider. Taking equations 6.1 to 6.3 into account, in detail we get

$$m^2VJ_y \cdot E = (\frac{\partial D}{\partial V} mg\sin\vartheta - \frac{\partial L}{\partial V} mg\cos\vartheta)M_\alpha + (\frac{\partial L}{\partial\alpha} mg\cos\vartheta - \frac{\partial D}{\partial\alpha} mg\sin\vartheta)M_V \qquad (6.8)$$

This can be rewritten to

$$m^2VJ_y \cdot E = -\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta) \cdot \left[M_\alpha + M_V \frac{\frac{\partial}{\partial\alpha}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)}{\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)} \right]$$

This results in:

$$E = \frac{-\frac{\partial}{\partial V}(L \cdot mg\cos\vartheta - D \cdot mg\sin\vartheta)}{m^2 V J_y} \cdot \left[M_\alpha + M_V \cdot \frac{\delta V}{\delta\alpha} \right] \qquad (6.9)$$

Under normal angles of attack ∂L/∂V·m·g·cosϑ - ∂D/∂V·m·g·sinϑ > 0, thus the requirement 6.7 is identical with the requirement

$$-\left[M_\alpha + M_V \frac{\delta V}{\delta\alpha} \right] \geq 0 \qquad (6.10)$$

δV/δα denotes the deviation of the speed by α, therefore the differential-quotient of the overall pitching-moment M derived by α and taken along the speed polar must be negative in order to achieve static longitudinal stability:

$$-\delta M/\delta\alpha > 0 \qquad (6.11)$$

This result is well in correspondence with those of chapter 5.

6.1 Fast pitching-oscillations

At stationary free flight gliding under condition 6.11 for static longitudinal stability, because of the requirements Δ V = Δϑ = 0 the characteristic equation 6.5 will be reduced to

$$J_y \cdot \lambda^2 - \overline{M}_{\dot\alpha} \cdot \lambda - M_\alpha = 0 \qquad (6.1.1)$$

This is the characteristic equation of a pitch-oscillation around the c.g. Since the attenuation factor $-M_{\dot\alpha}$ is always positive by nature, this equation provides real roots in case of stability with $-M_\alpha > 0$. By means of equations 6.3 we get

$$J_y \cdot \lambda^2 - \frac{\overline{c}}{V}(c_{m,\dot\alpha} + c_{m,\omega y}) \cdot q \cdot A \cdot \hat{c} \cdot \lambda - c_{m,\alpha} \cdot q \cdot A \cdot \hat{c} = 0 \qquad (6.1.2)$$

In order to solve this equation, next the derivatives herein have to be determined.

6.1.a The q-derivative

Dependent on the rotation of the glider around the lateral axis y with an angular speed $q = \omega_y$, the so-called q-derivatives, will play a roll. They result from the distinct air wash which emerges at the various parts of the glider by interference of the general airflow with speed V and of the local vertical air flow with speed $q \cdot r = \omega_y \cdot r$ of the rotation, and where r is the distance of the glider-part from the c.g. The change of the flow-direction thereupon then corresponds to an incremental angle of attack, also called "dynamic" angle of attack α_{dyn}, given by

$$\alpha_{dyn} = \text{atan}\,(\omega_y \cdot r / V) \approx \omega_y \cdot r / V \qquad (6.1.3)$$

Thereby at the elevator an incremental lift results which is given by

$$\Delta L_h = (c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h}{V} \cdot q_h \cdot A_h \qquad (6.1.4)$$

r_h is the distance of the aerodynamic centre of the elevator from the c.g., for the incremental lift coefficient follows

$$\Delta c_L = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot (c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h}{V} \qquad (6.1.5)$$

Taking into account that

$$c_{L,\omega y} = \partial c_L / \partial(\omega_y \cdot \hat{c} / V) \qquad 6.1.6)$$

the q-derivative of the elevator becomes

$$(c_{l,\omega y})_h = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \cdot (c_{l,\alpha})_h \qquad (6.1.7)$$

and for the overall derivative will result

$$c_{L,\omega y} = (c_{L,\omega y})_{wing+fuselage} + \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \cdot (c_{l,\alpha})_h \qquad (6.1.8)$$

The elevator contribution of the pitch attenuation moments $c_{m,\omega y}$ thus is

$$\begin{aligned} \Delta M_h &= -r_h \cdot \Delta L_h \\ &= -(c_{l,\alpha})_h \cdot \frac{\omega_y \cdot r_h^2}{V} \cdot q_h \cdot A_h \end{aligned} \qquad (6.1.9)$$

With $\Delta M_h = (c_{m,\omega y})_h \cdot \omega_y \cdot (\hat{c}/V) \cdot q \cdot A \cdot \hat{c}$ we get

$$(c_{m,\omega y})_h = -\frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (c_{l,\alpha})_h \qquad (6.1.10)$$

For the overall pitch-attenuation-moment it follows

$$c_{m,\omega y} = (c_{m,\omega y})_{wing+fuselage} - (c_{m,\omega y})_h \qquad (6.1.11)$$

At conventional glider-configurations, low sweep of the lifting wing, and proper elevator distance from c.g., usually the contributions of wing and fuselage are less than 1/10 of the elevator contribution. For most practical cases in model flying we can assume that the aerodynamic pressure at wing and elevator are about equal, $q_h/q \approx 1$, and thus we finally get:

$$c_{m,\omega y} \approx -\frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot (c_{l,\alpha})_h \qquad (6.1.12)$$

Using formula 4.16, this attenuation derivative can finally be written in the form

$$c_{m,\omega y} = -2\pi \cdot a_h \cdot a_{ph}(\alpha) \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \qquad (6.1.13)$$

This equation is of major importance for the design of a glider. The efficiency factor a_h pays attention to the geometric shape and to the aspect ratio Λ_h of the elevator, according to the extended lifting-line-theory with good approximation $a_h \approx \Lambda_h/(2+(\Lambda_h^2+4)^{1/2})$, e.g. for an elevator with $\Lambda_h = 6$ we roughly get $a_h \approx 0.72$. As described in chapter 4 the factor a_{ph} pays regard to the viscous flow-effects at the elevator-airfoil on its slope of the lift-coefficient with α. Usually the c.g of a glider is chosen close to optimum gliding or minimum sinkrate, then the lift at the elevator is close to zero and accordingly also its angle of attack. In order to increase the velocity of the glider, an increase of the angle of attack is required at the elevator. Roughly, most commonly used airfoils of elevators have symmetrical shape and their viscosity-factor at lower Re-numbers may deviate considerably from the ideal value $a_{ph} \approx 1$ for high speed. Thus, in order to guarantee a distinct pitch-attenuation-derivative $c_{m,\omega y}$ at any possible flight-velocity, for a_{ph} the minimum $a_{ph} \approx 1$ should be chosen and the parameters a_h, A_h and r_h^2 of the elevator accordingly be adapted. This means, for most practical purposes it is sufficient to use the equation

$$c_{m,\omega y} = -2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \qquad (6.1.13a)$$

In many cases the value of $c_{m,\omega y}$ can be adopted from models known to provide good attenuation behaviour.

6.1.b The ά - derivative of c_m

The attenuation-derivative by $\dot{\alpha} = d\alpha/dt$ on the one side takes the retarded new formation of the airflow from the lifting wing into account which results from the movement of the angle of attack at a disturbance, on the other side it considers to the downwash-fraction arriving at the elevator with delay after a non-stationary airflow-change at the wing. With change of the angle of attack by $\Delta\alpha$ in the first instance there will appear equal α-changes at wing and elevator. But only after a time delay Δt the downwash-change of the lifting wing becomes effective at the elevator what then leads to an incremental change of the angle of attack of the elevator. Under stationary flight-conditions $(\partial\alpha_w/\partial\alpha)\cdot\Delta\alpha$ corresponds to the relation of downwash and angle of attack. In a larger distance behind the lifting wing with quasi-elliptical wing-shape it can be assumed that

$$\frac{\partial \alpha_w}{\partial \alpha} \approx \frac{2}{\pi \cdot \Lambda} \cdot \frac{dc_{Lw}}{d\alpha} \qquad (6.1.15)$$

As shown in chapter 4, $c_{Lw,\alpha} = a_w \cdot a_{pw}(\alpha) \cdot 2 \cdot \pi$, wherein $a_{pw}(\alpha)$ takes care of the viscous airflow-effects in the boundary-layer of the airfoil used at the lifting wing. Thus we can write

$$\frac{\partial \alpha_w}{\partial \alpha} = \frac{4 \cdot a_w \cdot a_{pw}(\alpha)}{\Lambda} = \frac{4 \cdot a_w^\times}{\Lambda} \qquad (6.1.16)$$

For the case of non-stationary airflow *La Place*-transformation of the downwash-changes at the elevator yields with p=1/t

$$\Delta\alpha_w(p) = \frac{\partial \alpha_w}{\partial \alpha} \cdot \Delta\alpha(p) \cdot e^{-p \cdot \Delta t} \qquad (6.1.17)$$

and for the effective increment of the angle of attack at the elevator follows

$$\Delta\alpha_h(p) = \Delta\alpha(p) \cdot (1 - \frac{\partial \alpha_w}{\partial \alpha} \cdot e^{-p \cdot \Delta t}) \qquad (6.1.18)$$

Development into a series for small pitching-frequencies, $|p| << 1/\Delta t$, will yield

$$\Delta\alpha_h(p) \approx \Delta\alpha(p) \cdot (1 - \frac{\partial \alpha_w}{\partial \alpha}) + \frac{\partial \alpha_w}{\partial \alpha} \cdot p \cdot \dot{\alpha} \qquad (6.1.19)$$

Herewith the change of the lift at the elevator results to

$$\Delta L_h = (c_{l,\alpha})_h \cdot \Delta\alpha_h \cdot q_h \cdot A_h \qquad (6.1.19a)$$

$$\Delta c_{lh} = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot (c_{l,\alpha})_h \cdot \left[\Delta\alpha \cdot (1 - \frac{\partial \alpha_w}{\partial \alpha}) + \frac{\partial \alpha_w}{\partial \alpha} \cdot \Delta t \cdot \dot{\alpha} \right] \qquad (6.1.19b)$$

The first term in equation 6.1.19b corresponds to the stationary change, the second term corresponds to an ά –derivative, and taking into account the definition

$$c_{L,\dot{\alpha}} \equiv \partial c_L / \partial(\dot{\alpha} \cdot \hat{c} / V)$$

we will get

$$c_{L,\dot{\alpha}} = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{\partial \alpha_w}{\partial \alpha} \cdot (c_{l,\alpha})_h \cdot \Delta t \cdot \frac{V}{\bar{c}} \qquad (6.1.20)$$

Δt can be calculated by means of the speed V and the path r_h* how the changed wake has to travel from the lifting wing to the elevator, namely $\Delta t \approx r_h^*/V$. Thus the ά-derivative turns to be

$$c_{L,\dot{\alpha}} = \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h^\times}{\hat{c}} \cdot \frac{\partial \alpha_w}{\partial \alpha} \cdot (c_{l,\alpha})_h \qquad (6.1.21)$$

The momentum-derivative

$$c_{m,\dot{\alpha}} = \partial c_m / \partial(\dot{\alpha} \cdot \hat{c} / V)$$

can by means of the equation

$$\Delta M = -\Delta L_h \cdot r_h = \Delta L_h \cdot (r_h^\times - (X_{c.g.} - X_{Nw})) \qquad (6.1.22)$$

be written in the form

$$c_{m,\dot{\alpha}} = -\frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h^{\times 2}}{\hat{c}^2} \cdot \left(1 - \frac{X_{c.g.} - X_{Nw}}{r_h^\times}\right) \cdot \frac{\partial \alpha_w}{\partial \alpha} \cdot (c_{l,\alpha})_h \qquad (6.1.23)$$

he position of the aerodynamic centre of the wing, X_{Nw} , depends on the lift- and momentum-derivatives of the chosen airfoils according to

$$\frac{x_{Nw}}{\bar{c}} \approx \frac{1}{4}\text{T} \qquad (6.1.24)$$

For standard gliders $r_h^* \approx r_h$ and $q_h \approx q$. and thus with sufficient accuracy results

$$c_{m,\dot{\alpha}} \approx -\frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot \left(1 - \frac{X_{c.g.} - X_{Nw}}{r_h}\right) \cdot \frac{\partial \alpha_w}{\partial \alpha} \cdot (c_{l,\alpha})_h \qquad (6.1.24)$$

According to chapter 4, $c_{lh,\alpha} = a_h \cdot a_{ph}(\alpha) \cdot 2 \cdot \pi$, $a^X = a_w \cdot a_p(\alpha)$, and finally the attenuation derivative due to ά turns out to be

$$c_{m,\dot{\alpha}} \approx -2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot \left(1 - \frac{x_{c.g.} - x_{Nw}}{r_h}\right) \cdot \frac{\partial \alpha_w}{\partial \alpha} \qquad (6.1.25)$$

Even at major changes of X_{Nw} due to viscous airfoil-effects for most standard-gliders $|X_{c.g.} - X_{Nw}| << r_h$ for the non-critical **α**-region of the wing-sections therefore with sufficient accuracy we can assume that

$$c_{m,\dot{\alpha}} \approx -2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot \frac{\partial \alpha_w}{\partial \alpha} \qquad (6.1.25)$$

Altogether the attenuation-derivatives will approximately supply

$$c_{m,\dot{\alpha}} \approx c_{m,\omega y} \approx -2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \qquad (6.1.26)$$

According to formula 6.1.16 $\partial\alpha_w/\partial\alpha$ is of the order of magnitude of $4 \cdot a_w^* / \Lambda_w$, and consequently for gliders with higher aspect ratios of the lifting wing the downwash-derivative $\partial\alpha_w/\partial\alpha$ may be neglected without major error. Thus, finally it can be stated that the major contribution to the attenuation of the rotational movement of a glider result from the q-derivative.

6.1.c The α - derivative of c_m

The lift-dependence of a glider on the angle of attack within the non-critical α-range of the chosen airfoils in a first approach is composed of shares from the lifting wing and the elevator. The influence of the fuselage shall here be neglected. Since effects resulting from drag are also of secondary importance, from chapter 2 and with $\cos\alpha_w \approx 1$ and $|D_h \cdot \sin\alpha_w| << |L_h \cdot \cos\alpha_w|$ the total lift of the glider is given by

$$L = L_w + L_h \, , \qquad (6.1.27)$$

and using the expression with lift coefficients

$$L = c_L \cdot q \cdot A, \qquad L_w = c_{Lw} \cdot q \cdot A_w, \qquad L_h = c_{lh} \cdot q \cdot A_h$$

we get

$$c_L = c_{Lw} + \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot c_{lh} \qquad (6.1.28)$$

Taking into account the downwash-factor $\partial\alpha_w/\partial\alpha$, the overall lift slope of the glider results to be

$$\boxed{c_{L,\alpha} = (c_{L,\alpha})_w + (1 - \frac{\partial\alpha_w}{\partial\alpha}) \cdot (c_{l,\alpha})_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A}} \qquad (6.1.29)$$

By use of formula 6.1.29 the α-derivative of the pitching-moment turns out to be

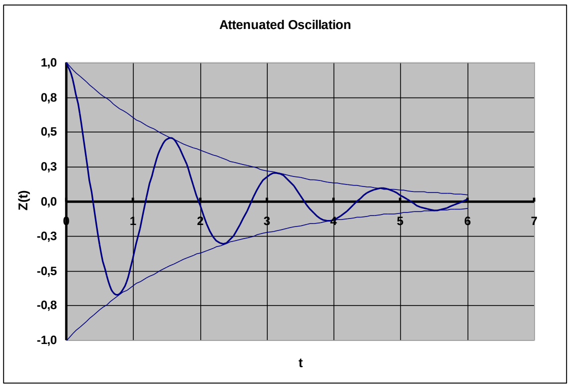
$$\boxed{c_{m,\alpha} = (c_{L,\alpha})_w \cdot \frac{X_{c.g.} - X_{Nw}}{\hat{c}} + (1 - \frac{\partial\alpha_w}{\partial\alpha}) \cdot (c_{l,\alpha})_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}}} \qquad (6.1.30)$$

This derivative essentially depends on the size of the elevator and its distance from c.g. The position of the aerodynamic centre of the wing is influenced by the viscous effects of the chosen airfoils and given by equation 6.1.24.

By use of equations 4.15 and 4.16 and under the assumption that $q_h \approx q$ finally follows

$$\boxed{c_{m,\alpha} = 2\pi \cdot a_w^{\times} \frac{X_{c.g.} - X_{Nw}}{\hat{c}} + (1 - \frac{\partial\alpha_w}{\partial\alpha}) \cdot 2\pi \cdot a_h^{\times} \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}}} \qquad (6.1.31)$$

6.1.d Consequences for the Fast Pitching-Oscillations



Generally, the characteristic equation of an oscillation is written in the form

$$\boxed{\lambda^2 + 2 \cdot \delta \cdot \lambda + \omega_o^2 = 0} \qquad (6.1.32)$$

therein ω_o [s^{-1}] is called circular "eigen"-frequency and δ [s^{-1}] is called attenuation constant. Oscillation is given for the case when $\delta < \omega_o$.

$$0 \equiv \begin{vmatrix} m \cdot \lambda - X_V & -X_\vartheta \\ -Z_V & m \cdot V \cdot \lambda - Z_\vartheta \end{vmatrix}$$

In this case equation 6.1.32 has two conjugated complex solutions

$\lambda_{1;2} = -\delta \pm j\omega$ with $\omega = \omega_o^2 - \delta^2$ (j denotes the imaginary unit).

A disturbance Z(t) then results from the general solution

$$Z(t) = (C_1 \cdot e^{j\omega t} + C_2 \cdot e^{-j\omega t}) \cdot e^{-\delta \cdot t} \qquad (6.1.33a)$$

At t = 0, $Z = Z_o = Z(t_o)$ for the undetermined coefficients C_1 and C_2 results $Z_o = C_1+C_2$, and for our purposes Z(t) can finally be transformed into equation

$$Z(t) = Z(t_o) \cdot e^{-\delta \cdot t} \cdot cos(\omega \cdot t + \varphi) \qquad (6.1.33)$$

Above graphic illustrates the attenuation of a disturbance Z(t) with time, the two enveloping curves describe the time dependent damping of the oscillatory motion after the disturbance.

The larger the attenuation constant δ, the more rapidly the envelope $Z(t_o) \cdot exp(-\delta \cdot t)$ approaches 0. The usual measure for it is $D = \delta/\omega_o$.

By comparison of equation 6.1.2 with equation 6.1.32, for the oscillations of the pitching movement of the glider after disturbance of the stationary gliding we get

$$\delta = -\frac{1}{2 \cdot J_y} \cdot \frac{\hat{c}}{V} \cdot (c_{m,\dot{\alpha}}) \cdot q \cdot A \cdot \hat{c} \qquad (6.1.34)$$

Taking into account equation 6.1.26 we will finally get

$$\boxed{\delta = \frac{\pi}{2J_y} \cdot a_h^{\times} \cdot A_h \cdot r_h^2 \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \cdot \rho \cdot V} \qquad (6.1.35)$$

Herein ρ is the density of the air.

- This equation tells us that after disturbance of the angle of attack and/or the gliding angle, the damping of pitching-oscillations mainly depends on shape and geometry of the elevator and in particular most strongly on the distance of the elevator from c.g. since this act with the second power.

- In the non-critical α-range usually the viscosity factor of the elevator airfoil $a_{ph}(\alpha) \geq 1$, in particular at very low Re-numbers where viscosity-effects in the boundary layer of the airfoil play a considerable role for the airflow. In order to make sure that a glider will provide desired attenuation, the lower limit $a_{ph}=1$ should be chosen in equation 6.1.35, and correspondingly the other parameters for the required δ.

- As mentioned earlier, attention to the downwash is paid by $\partial\alpha_w/\partial\alpha \approx 4 \cdot a_w^{\times}/\Lambda_w$. In principle it will become smaller with increasing aspect-ratio of the lifting wing, and as will still be discussed later, due to deteriorating viscous-effects with increasing flight velocity it will decrease with increase of the velocity. At lower velocity, for gliders with small aspect-ratio ($\Lambda_w \approx 10$) the downwash factor may become $\partial\alpha_w/\partial\alpha \approx 0.5$, at higher speed for gliders with higher aspect ratio (($\Lambda_w \approx 25$) the lower border will be in the range of $\partial\alpha_w/\partial\alpha \approx 0.15$. This means that the attenuation of the pitching-oscillation will become smaller with increasing aspect-ratio of the lifting wing which has to be taken into account for the size and the momentum-arm of the elevator.

We also learn from equation 6.1.35 that the attenuation of the pitching oscillation increases with the speed and with the mean aerodynamic chord (MAC) of the glider.

- A factor to which most often not sufficient attention is drawn is the mass-moment of inertia J_y around the lateral axis of the glider. According to equation 3.4. the masses of the tail and the nose of the glider contribute most to this moment, thus in order to achieve proper attenuation and to keep the elevator dimensions small, according to equation 6.1.35 a construction goal should be to keep the elevator mass as low as possible (correspondingly the mass in the front part of the fuselage can be reduced).

- The attenuation of the pitching oscillation, however, must not be chosen too strong because on the other side the response to the elevator control-panel may become too slow for the necessary manoeuvrability. When designing a new glider mostly it can be very helpful to determine the values of the characteristic parameters for the attenuation from gliders known to provide the required δ-measure.

For the circular "eigen"-frequency ω_0 of the corresponding non- oscillation it matters

$$\omega_o^2 = -\frac{1}{J_y} \cdot c_{m,\alpha} \cdot q \cdot A \cdot \hat{c} \qquad (6.1.36)$$

and taking into account equation 6.1.31 it will turn out to become

$$\omega_o^2 = -\frac{1}{J_y} \cdot \left(2\pi \cdot a_w^{\times} \frac{X_{c.g.} - X_{Nw}}{\hat{c}} - (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot 2\pi \cdot a_h^{\times} \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \right) \cdot q \cdot A \cdot \hat{c} \qquad (6.1.37)$$

- Since the aerodynamic centre of the lifting wing is determined by the wing-design and the chosen airfoils and the position of the c.g. in principle results from the requirements for optimum gliding and minimum sinkrate, and since all other parameters are determined by the requirement for sufficient static stability and attenuation, there is no further possibility to affect ω_0.

6.2 Slow Oscillations of the Centre of Gravity

At instationary longitudinal motion of a glider with constant angle of attack, $\Delta\alpha \equiv 0$, according to *F.W. Lancaster* a so called "pitch-phugoid" develops after disturbance in V and ϑ. This usually is a long-period mode in which the c.g. carries out a lightly damped oscillation along its stationary flight path. It involves a slow oscillation over many seconds in which energy is exchanged between vertical and forward velocity (potential and kinetic energy). (The equations of motion now just provide information on the angle of the elevator control necessary to maintain a constant angle of attack.)

The relations between the weight G = m·g, the lift L, the drag D of the plane, and the gliding angle ϑ are to be derived from equations 6.1 and 6.2 of the forces in the directions of the x- and z-axes. When setting $\Delta\alpha \equiv 0$ we receive

$$m \cdot \dot{V} = X_V \cdot \Delta V + X_\vartheta \cdot \Delta\vartheta \qquad (6.2.1)$$

$$m \cdot V \cdot \dot{\vartheta} = Z_V \cdot \Delta V + Z_\vartheta \cdot \Delta\vartheta \qquad (6.2.2)$$

Thereupon the characteristic equation of the c.g.-phugoid can be written in the form

$$m^2 \cdot V \cdot \lambda^2 + (-m \cdot G \cdot \sin\vartheta + \frac{\partial D}{\partial V} \cdot m \cdot V) \cdot \lambda + (-\frac{\partial D}{\partial V} \cdot G \cdot \sin\vartheta + \frac{\partial L}{\partial V} \cdot G \cdot \cos\vartheta) = 0 \qquad (6.2.4)$$

Since L = L(V^2) and D = D(V^2), it follows

$$\partial L/\partial V = 2 \cdot L/V \quad \text{and} \quad \partial D/\partial V = 2 \cdot D/V$$

and with L = G·cosϑ , D = G·sinϑ we get

$$m^2 \cdot V \cdot \lambda^2 + (-m \cdot G \cdot \sin\vartheta + 2 \cdot m \cdot G \cdot \sin\vartheta) \cdot \lambda + (-\frac{2}{V} \cdot G^2 \cdot \sin^2\vartheta + \frac{2}{V} \cdot G^2 \cdot \cos^2\vartheta) = 0 \qquad (6.2.5)$$

Thus, finally the characteristic equation can be written in the form

$$\lambda^2 + \frac{g \cdot \sin \vartheta}{V} \cdot \lambda + \frac{2 \cdot g^2}{V^2} \cdot (\cos^2 \vartheta - \sin^2 \vartheta) = 0 \qquad (6.2.6)$$

For smaller gliding angles $\sin^2 \vartheta \approx 0$.

6.2.a Consequences for the slow c.g.-oscillations

Like for the fast-pitching oscillations, the general characteristic equation of the damped c.g.-oscillation is to be written in the form

$$\lambda^2 + 2 \cdot \delta \cdot \lambda + \omega_o^2 = 0 \qquad (6.2.7)$$

Therein ω_o [t^{-1}] is called circular "eigen"-frequency and δ [t^{-1}] is the damping constant. Oscillation is given when $\boldsymbol{\delta < \omega_o}$. Like for the characteristic equation of the fast pitch oscillation, then there will exist two solutions λ_3 and λ_4

$$\lambda_{3;4} = -\delta \pm j\omega \quad \text{with} \quad \omega = \omega_o^2 - \delta^2$$

This again leads to a damped oscillating disturbance. Comparison of equation 6.2.7 with equation 6.2.6 yields:

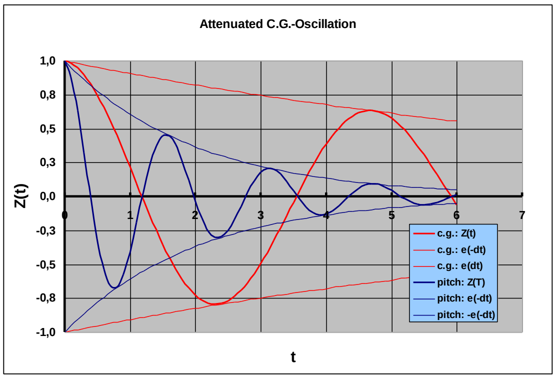
Damping constant:

$$\delta = \frac{g \cdot \sin \vartheta}{2 \cdot V} \qquad (6.2.8)$$

"Eigen"-frequency:

$$\omega_o = \frac{g}{V} \cdot \sqrt{2 \cdot (\cos^2 \vartheta - \sin^2 \vartheta)} \approx \sqrt{2} \cdot \frac{g \cdot \cos \vartheta}{V} \qquad (6.2.9)$$

Thus, damping and "eigen"-frequency only depend on the velocity V and on the gliding angle ϑ of the corresponding stationary flight-state. They do not depend on the characteristics of a given glider.



The left graphic provides a rough idea of the difference between the fast pitching-oscillations and the slow, damped c.g. oscillations.

The subsequent examples will provide the relations as they are observed in flight practice.

6.3 Coupled Pitch- and C.G.-Oscillations

In some cases, it may be desired to consider the equations of motion for a concurrent disturbance in velocity, gliding angle and angle of attack. In this case the characteristic equation $F_4(\lambda)$ (equation 6.5 and 6.6)) has to be solved. The coupling of fast pitch- and slow c.g.-oscillations will cause a certain shift of the roots λ_1 to λ_4. As before $\lambda_{1;2}$ may be the roots of the fast pitching-motion and $\lambda_{3;4}$ those of the slow phugoid-motion. Using the designation of the coefficients of F_4 as in equation 6.6 the mathematic evaluation provides following equations for the four roots:

$$\lambda_1 + \lambda_2 = -B - (\lambda_3 + \lambda_4) \qquad (6.3.1a)$$

$$\lambda_1 \cdot \lambda_2 = C - \lambda_3 \cdot \lambda_4 - (\lambda_1 + \lambda_2) \cdot (\lambda_3 + \lambda_4) \qquad (6..3.1b)$$

$$\lambda_3 \cdot \lambda_4 = E / (\lambda_1 \cdot \lambda_2) \qquad (6.3.1c)$$

$$\lambda_3 + \lambda_4 = -(D + (\lambda_1 + \lambda_2) \cdot \lambda_3 \cdot \lambda_4) / (\lambda_1 \cdot \lambda_2) \qquad (6.3.1d)$$

For the initial approximation

$$(\lambda_3 + \lambda_4)^{(0)} = 0 \quad \text{and} \quad (\lambda_3 \cdot \lambda_4)^{(0)} = 0$$

as a first solution will be achieved:

$$(\lambda_1 + \lambda_2)^{(1)} = -B \qquad (6.3.2a)$$

$$(\lambda_1 \cdot \lambda_2)^{(1)} = C \qquad (6.3.2b)$$

$$(\lambda_3 \cdot \lambda_4)^{(1)} = E / C \qquad (6.3.2.c)$$

$$(\lambda_3 + \lambda_4)^{(1)} = (-D \cdot C + B \cdot E) / C2 \qquad (6.3.2.d)$$

In a second step then the roots λ_1 to λ_4 can be determined and the corresponding motion investigated. This will not further be followed up in this context.

For RC-controlled airplanes it is rather important that the fast pitching-oscillation is sufficiently damped since otherwise the pilot will not be able to correct these disturbances. On the other hand, to correct slow phugoidic oscillations does usually not cause problems. According to experience for most standard gliders the damping of the pitch-disturbances is such that the flight behaviour after small disturbances can be predicted by means of the separated characteristic equations of motion for fast pitching-oscillations and slow phugoidic movement. It may be important to solve the coupled characteristic equation for “wing only”-gliders with minor degree of longitudinal attenuation in order to predict their flight stability.

6. Examples of Proven Gliders

7.1 Assessment of the Mass Moment of Inertia, J_y

According to chapter 3 the mass moment of inertia around the lateral airplane-axis is given by the equation

$$J_y = \sum_i m_i \cdot r_i^2$$

Inertia forces derive from the attribute of the mass to resist accelerations. The mass of rotational accelerations is represented by mass moment of inertia terms J. The total mass moment of inertia related to the rotation of a glider around the lateral y-axis through the c.g., Jy, results from the various parts of the glider: the lifting wing, the fuselage, and the tail-parts (fin and elevator, or V-tail).

An approximate value of J_y can be assessed for most gliders according to the approach

$$J_y = m_w \cdot r_w^2 + m_{f,f} \cdot r_{f,f}^2 + m_{f,r} \cdot r_{f,r}^2 + m_t \cdot r_t^2 \qquad (7.1.1)$$

Therein m_w denotes the mass of the lifting wing, r_w the distance of the c.g. from the mass centre of the wing, $m_{f,f}$ the share of the fuselage-mass in front of the c.g., $r_{f,f}$ the distance of the c.g. from the mass-centre in the front of the fuselage, $m_{f,r}$ the mass of the rear-tube of the fuselage behind the c.g., $r_{f,r}$ the distance of the c.g. from the mass-centre of the rear-fuselage part, m_t the mass of the tail and r_t the distance of the c.g. from the mass-centre of the tail.

Later on, two examples will be given. One of them will be that of an **F3J-glider** with 3.7-meter wingspan and a mass of approximately 2.3 kg. For this model it was theoretically estimated that

m_w = 1.30 kg, r_w = 0.03 meter

$m_{f,f}$ = 0.68 kg, $r_{f,f}$ = 0.4 meter

$m_{f,r}$ = 0.28 kg, $r_{f,r}$ = 0.7 meter

m_t = 0.12 kg, r_t = 1.15 meter

Herewith the mass-moment of inertia was expected to become

$$J_y = 1.30 \cdot 0.03^2 + 0.68 \cdot 0.4^2 + 0.2 \cdot 0.7^2 + 0.12 \cdot 1.15^2 \ \text{kg} \cdot \text{m}^2$$

$$= 0.0012 \quad + 0.109 \quad + 0.098 \quad + 0.159 \ \text{kg} \cdot \text{m}^2$$

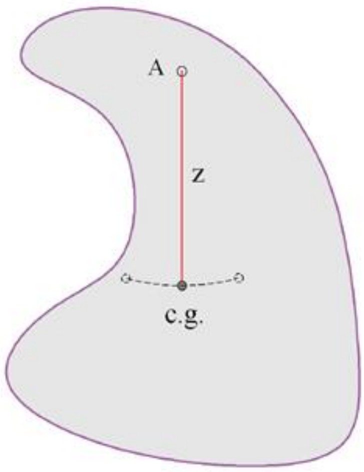
$$= 0.367 \ \text{kg} \cdot \text{m}^2$$

We see that the smallest contribution results from the lifting wing because its mass-centre is rather close to the c.g., whilst the largest contribution results from the tail-part which has the lowest mass, but its distance from the c.g. is the largest.

Generally, in order to keep the mass-moment of inertia small, as desired by the damping-requirements, at standard-gliders the weight of the tail should be kept as low as possible. Each gram saved at the tail also reduces the balancing-ballast in the nose of the fuselage about factor 2 to 3 and correspondingly also the mass-moment of inertia of the fuselage-front.

- As was shown in chapter 6, J_y plays an important roll for the attenuation of the fast-pitching oscillations, see equation 6.1.35. With increasing value of J_y in general the size A_h or the momentum arm r_h of the elevator have to be increased to compensate for. If also a certain static stability is required according to chapter 4, equation 4.1.7., the right balance between A_h and r_h has to be found.

7.2 Experimental Determination of the Mass -Moment of Inertia, Jy



A simple practical method to determine the mass-moment of inertia of any given body is the following; If a body like that in the left graphic is equipped with an axis through A it can be stimulated to swing around this axis and the time T for one full period of oscillations is given by

$$T \approx 2 \cdot \pi \cdot \sqrt{J/m \cdot g \cdot z} \qquad (7.2.1)$$

Herein J is the mass-moment of inertia related to the axis through A, z is the distance from the c.g. According to physical mechanics J can also be described in the form

$$J = J_{c.g.} + m \cdot z^2 \qquad (7.2.2)$$

where $J_{c.g.}$ is the mass-moment of inertia for the body related to the axis through the centre of gravity, parallel to the axis through A.

Combining the two equations we get

$$J_{c.g.} \approx \left(\frac{T}{2 \cdot \pi}\right)^2 \cdot m \cdot g \cdot z - m \cdot z^2 \qquad (7.2.3)$$

$\omega \approx 2\pi/T$ is the oscillation-frequency of this swinging of the pendulum.

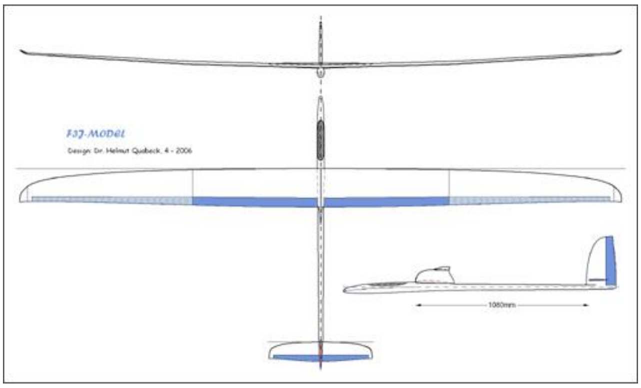
By means of this pendulum-method for a given model-airplane the mass moment of inertia J_y around the lateral y-axis through the c.g. can easily be determined.

For example, when the F3J-Model given in section 7.2 was hung up with nose down at the end of the fuselage it swings with a period-time T = 2.32 s. With a distance of the swinging-axis from the c.g., z = 1.2 meters, by means of formula 7.2.3 one yields

$$J_y = (2.32/2\pi)^2 \cdot 2.3 \cdot 9.81 \cdot 1.2 - 2.3 \cdot 1.2^2 = 0.38 \text{ kg}\cdot\text{m}^2$$

The above theoretical estimate of 0.367 $\text{kg}\cdot\text{m}^2$ differs not much from the practical result. In order to determine the appropriate values of the geometric parameters of the model for proper static stability and attenuation of the fast-pitching oscillations it was a good guide.

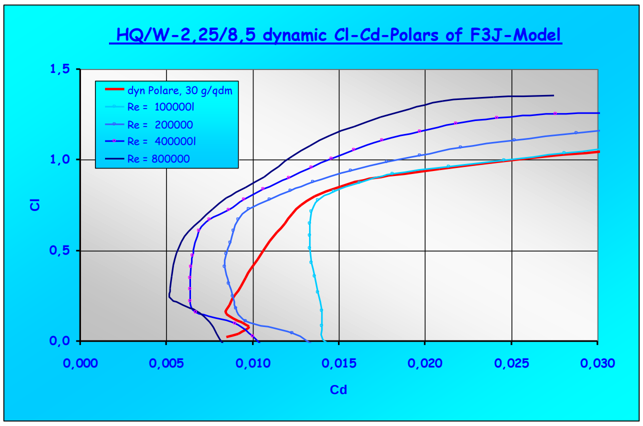
7.3 Example of an F3J-Model



The left graphic shows the 3 side-draft for a new **F3J**-model designed by the author. Flight-mechanical characteristics of the model as given below have been determined by means of the ***"FMFM"***-program (*Flight-Mechanics for Flight-Models*) which is described in more detail on the homepage www.hq-modellflug.de.

Major goals for the model were superior sinkrate and gliding-performance under all flight conditions, as well as proper flight-stability and manoeuvrability as required in F3J-contests.

a. Since the lifting wing is mainly responsible for the performance of a glider-model, major attention has been paid to its geometric design and its aerodynamic characteristics such as lift-efficiency, airfoil- and induced drag. The airfoils finally chosen are the *"HQ/W-2.25/8.5"* for the whole lifting wing, and the *"HQ/W-0/9* for elevator and fin. The distribution of the wing chord was chosen such that the lift-distribution of the model was close to ideal. According to good practical experience, stall problems at the lifting wing have been solved by appropriate wingtips.



b. Usually the selection of distinct airfoils for a lifting wing is done by a comparison of the performance of potential airfoils over the possible speed range given by the weight/unit-area. For manned gliders this at the end is given in form of a quasi-stationary velocity polar. In the first instance it requires that for all possible stationary velocities of the glider the corresponding lift, the airfoil-drag, the drag caused by interference of the airplane parts, and the induced drag must be determined for the lifting wing. In the left polar-graphic of the *"HQ/W-2,25/8,5* this particular quasi-stationary polar is indicated by the red polar curve.

Other c_l-c_d-values then those on the red quasi-stationary polar may be reached under instationary flight conditions, such as given at a fast turn or a loop, however, this is of minor importance for the performance-considerations.

The stationary gliding- and sink-velocities of a glider are given by

$$V = \sqrt{\frac{2}{\rho} \cdot \frac{m \cdot g}{A} \cdot \frac{\cos \vartheta}{c_L}} \approx 4 \cdot \sqrt{\frac{m}{A} \cdot \frac{\cos \vartheta}{c_L}} \tag{7.3.1}$$

$$V_z = \sqrt{\frac{2}{\rho} \cdot \frac{m \cdot g}{A} \cdot \frac{c_W^2}{c_L^3} \cdot \cos^3 \vartheta} \approx 4 \cdot \sqrt{\frac{m}{A} \cdot \frac{c_W^2}{c_L^3} \cdot \cos^3 \vartheta} \tag{7.3.2}$$

The corresponding stationary gliding number is given by

$$G.N. = 1/\tan\vartheta = L / D = c_L / c_D \tag{7.3.3}$$

In order to determine the potential performance of a given lifting wing with chosen wing-sections the author usually ascertains the functional dependence of the wing only sink-rates and gliding numbers given by

$$S.R._w = c_{Dw}^2 / c_{Lw}^3 \tag{7.3.4}$$

$$G.N._w = c_{Lw} / c_{Dw} \tag{7.3.5}$$

The drag related to the chosen lifting-coefficient results from the properties of the airfoil and from the free vortices, in total we have $c_{Dw} = c_{Dp} + c_{Di} \approx c_{Dp} + c_{Lw}^2/\pi\Lambda_w$.

For the planned F3J-model the optimum working point for best thermal soaring is around $c_l = 0.9$. Next, we will see where the c.g. must then be located in order to achieve this working point whilst flying.

c. Having determined the optimum c_{Lw}-c_{Dw}-working point of the lifting wing for slow performance gliding, next the position of the centre of gravity c.g. which enables the glider to achieve these optimum flight conditions while soaring has to be found. According to flight dynamics, with c_{Mf} denoting the momentum-coefficient of the fuselage, and assuming that the momentum coefficient of the elevator can be neglected, the longitudinal momentum equation for the centre of gravity generally yields.

$$\frac{X_{c.g.}}{\hat{c}} = \frac{X_{Nw}}{\hat{c}} + \frac{1}{c_{Lw}} \left(c_{Lh} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} - c_{Mow} - c_{Mf} \right) \tag{7.3.6.a}$$

For the thinner F3J-fuselages c_{Mf} can be neglected, thus for zero lift at the elevator it turns out

$$\frac{X_{c.g.}}{\hat{c}} = \frac{X_{Nw}}{\hat{c}} - \frac{c_{Mow}}{c_{Lw}} \tag{7.3.6.b}$$

In cases like the one being considered where the same profile is used in all wing sections, the momentum coefficient c_{Mow} closely corresponds to that of the profile. The lift-derivative will be determined for the hole wing, taking into account the local values of chord, sweep, and twist. Next step in the design-routine of a plane will usually be to determine the dimensions and aerodynamic features of the elements for longitudinal flight control and stability. For a normal glider these elements are the rear fuselage-part behind the c.g. and the elevator. (For wing only models S-shaped profiles (with positive momentum), sweep, and negative twist of the lifting wing will take over the longitudinal stability functions, but this case will be discussed in another paper.)

The static stability for sure is most important for the longitudinal flight-stability of an airplane, as shown in chapter 5:

$$\sigma = (X_N - X_{c.g.}) / \hat{c}$$

From many glider constructions the author got the feedback, that in according to the conclusions of this article the longitudinal stability of F3J-gliders should at least be larger than 0.15:

$\sigma \geq 0.15$!

Consequently, for the elevator and its momentum arm related to the c.g. in according to chapter 4 they have to be sized such that the aerodynamic centre of the total airplane fulfils the requirement

$$\Delta X_N / \hat{c} \geq 0.15 + \Delta X_{c.g.} / \hat{c}$$

In order to be on the safe side concerning longitudinal flight stability, in chapter 4 the final equation proposed for the dependence of the aerodynamic centre on the wing- and elevator-characteristics was

$$\frac{\Delta X_N}{\hat{c}} = \frac{a_h/a_w \cdot A_h/A}{1 + a_h/a_w \cdot A_h/A} \cdot \frac{r_{Nh}}{\hat{c}}$$

Neglecting the downwash given by $\partial c_{Lw}/\partial\alpha \approx 0$ for the lower positioned elevator of the F3J-model this equation provides a value for r_{Nh}, which guarantees the desired sufficient longitudinal flight stability.

d. As in section 6.1.d was deduced, the attenuation-constant **δ** of fast pitching-oscillations is affected by various parameters given in equations 6.1.34/35. Written in a more practical form we get

$$\delta = -\frac{q \cdot A \cdot \hat{c}^2}{2 \cdot V} \cdot \frac{c_{m,\dot{\alpha}}}{J} = -\frac{\rho \cdot V \cdot A \cdot \hat{c}^2}{4} \cdot \frac{1}{J} \cdot \left\{-2 \cdot \pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2} \cdot \left(1 + \frac{\partial a_w}{\partial a}\right)\right\}$$

- First, we see that the damping of the oscillations increases with the soaring velocity V. Thus, minor influences due to no-viscous airfoil effects which are reflected by the factors a_{pw} and a_{ph} are generally overwhelmed with increasing velocity.

- Secondly, for a given wing the major parameters by which attenuation can be influenced are the mass-moment of inertia J_y and size A_h, the shape (factor a_h), and the momentum arm r_h of the elevator.

For a lifting wing the α-derivative of the downwash far behind the wing is given by $\partial\alpha_w/\partial\alpha \approx 4 \cdot a_w^\times / \Lambda_w$. Thus, it decreases with the aspect-ratio Λ_w of the lifting wing. With data given earlier the α-derivative for the planned F3J-Glider ranges according to $0.15 \leq 4 \cdot a_w^\times / \Lambda_w \leq 0.25$ and cannot be influenced by the elevator characteristics.

Consequently, the only remaining design-element for proper damping of disturbances is the ratio of the q-derivative and the mass-moment of inertia

$$\frac{c_{m,\omega y}}{J_y} = -\frac{1}{J_y} \cdot 2 \cdot \pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2}$$

From practical experience with various F3J-models and analyses of successful other F3J-models, the author has found that appropriate dynamic damping is achieved when this ratio ranges within -30 to -40.

- As was laid out in section 7.1, for a lower-weight F3J-model the mass-moment of inertia is around J_y = 0.4 kg·m². However, when a model is being build it may easily happen that the weight of the tail gets higher than desired and, since the mass of the tail contributes most to J_y, a minor damping than planned will appear. Thus, in order to be on the safe side concerning dynamic longitudinal damping, it may often be better to assume that $J_y \approx 0.5$. Since damping increases with flight-velocity, the viscous airstream-effects at the elevator are in fact only important for flight-conditions near the working point c_l = 0.9.

e. For the F3J-model shown above the following model-parameters based on ***EPPLER***-analyses were chosen:

Mean chord	$\hat{c}$ =	209.54 mm
Lifting-area of the wing	A =	0.704 m²
Aspect ratio of the wing	Λ_w =	17.41

Lift-efficiency of the wing a_W = **0.897**

Airfoil of the wing *HQ/W-2,25/8,5*

Mean chord of the elevator $\hat{c}_h$ = 101.5 mm

Lifting-area of the elevator A_h = 0.065 m2

Lift-efficiency of the elevator a_h = 0.76

Airfoil of the elevator *HQ/W-0/9* (Nowadays the ***HQ/ACRO-0/12*** would be chosen!)

Further

Mass moment of inertia $J_y = 0.367\ kg \cdot m^2$

Therefrom the dynamic stability is calculated to be

Dynamic stability measure: $c_{m,\omega y}/ J_y \approx -24{,}2!$

which is close to the required stability range.

Taking into account the viscous effects of the airstream around the lifting wing, above we had found that in order to achieve the working point conditions around c_l = 0.9 with cMo = -0,09 and c_{Lw} 0 0,097 the position of the centre of gravity should be chosen at

Centre of gravity: $X_{c.g.}/\hat{c}$ = 0.347

The chosen length of the momentum arm between c.g. and the aerodynamic centre of the elevator shall be

Length of momentum arm: r_h =1.032 m

then sufficient static longitudinal flight-stability will be given. As shown before, the overall aerodynamic centre of the model is determined by the formula

$$\frac{\Delta X_N}{\hat{c}} \approx \frac{a_h / a_W \cdot A_h / A_W}{1 + a_h / a_W \cdot A_h / A_W} \cdot \frac{r_{Nh}}{\hat{c}}$$

And finally, the static longitudinal static stability-measure for the chosen position of the c.g. corresponding to the working point **c_l = 0.9** turns out to be

Static stability : $\sigma = (X_N - X_{c.g.})/\hat{c} \approx 0.15$

According to experience in flight practice this is a good stability-value for the planned F3J-Model!.

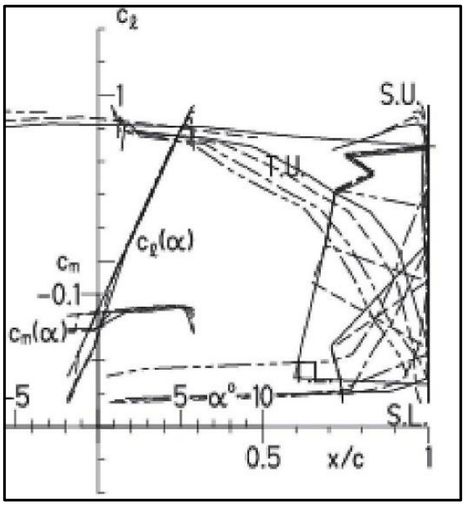
Th graphic below shows the c_l-α-polars of the wing section HQ/W-2,25/8,5 for low speed, analysed with Eppler *PROFIL06*.. Different from the values gained from the *X-Foil* analyses the optimum sinkrate is achieved for c_l = 0,85, and the value of the corresponding α-derivative is $c_{l,\alpha}$ = 0,107, close to non-viscous profile behaviour. Thus, with apw ≈ 0.97 and aph =1.1 results

Centre of gravity: $X_{c.g.}/\hat{c}$ = 0,25 + cmo/cl = 0.25 + 0.09/0,76 = 0,369

Aerodynamic centre of the glider: $X_N/\hat{c} \approx 0{,}566$

Static stability : $\sigma = (X_N - X_{c.g.})/\hat{c} \approx 0.197$

Using the Eppler-***PROFILE06***-program, for this F3j-modell the according Centre of gravity provided optimum performance at slow stationary gliding, no further trimming with ballast was necessary.



Meanwhile for quite a number of own larger and smaller glider-constructions, and on request for many other modellers the Eppler-analyses always provided C.G.s close to best practical performance.

f. Further, it will be of interest to which degree fast pitching-oscillations of the F3J-glider will be affected by the flight-mechanical characteristics calculated before. In chapter 6.1.d we had shown that a disturbance of the angle of attack is described by equation 6.1.33

$$Z(t) = Z(t_o) \cdot e^{-\delta \cdot t} \cdot cos(\omega \cdot t + \varphi)$$

Wherein $\omega = \omega_o^{\,2} - \delta^2$ can be calculated according to

$$\omega_o^2 = -\frac{1}{J_y} \cdot \left(2\pi \cdot a_w \frac{X_{c.g.} - X_{Nw}}{\hat{c}} - (1 - \frac{\partial \alpha_w}{\partial \alpha}) \cdot 2\pi \cdot a_h \cdot \frac{q_h}{q} \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \right) \cdot q \cdot A \cdot \hat{c}$$

$$\delta = \frac{\pi}{2J_y} \cdot a_h \cdot A_h \cdot r_h^2 \cdot (1 + \frac{\partial \alpha_w}{\partial \alpha}) \cdot \rho \cdot V$$

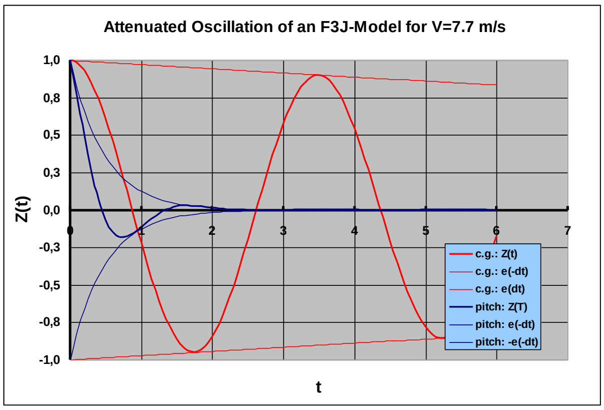
"Eigen"-frequency of pitch-oscillations	$\omega_o = 0.63 \cdot V$ [s^{-1}]
Damping constant of pitch-oscillations	$\delta = 0.34 \cdot V$ [s^{-1}]
Frequency of pitch-oscillations	$\omega = (\omega_o^2 - \delta^2)^{1/2} = 0.53 \cdot V$ [s^{-1}]

Oscillation-frequency and attenuation increase with increasing speed, thus the worst flight-state concerning damping of pitch-disturbances is given for conditions related to the working-point where the flight-velocity is at its minimum

Velocity at optimum working-point **V** $\approx 4 \cdot ((m/A)/(a_w \cdot c_a(0.9))^{1/2} = 8.3$ m/s

Here we have $\boldsymbol{\omega_o} = 5{,}23$ s^{-1}, $\boldsymbol{\delta} = 2.82$ s^{-1}, $\boldsymbol{\omega} = 4{,}40$ s$^{-1.}$

For the slow c.g.-oscillations it was shown in chapter 6.2.a that



$$\omega_{o,c.g.} \approx \sqrt{2} \cdot \frac{g \cdot \cos \vartheta}{V}$$

$$\delta_{c.g.} = \frac{g \cdot \sin \vartheta}{2 \cdot V}$$

With $\vartheta \approx 2.7°$ and V = 7.7 m/s

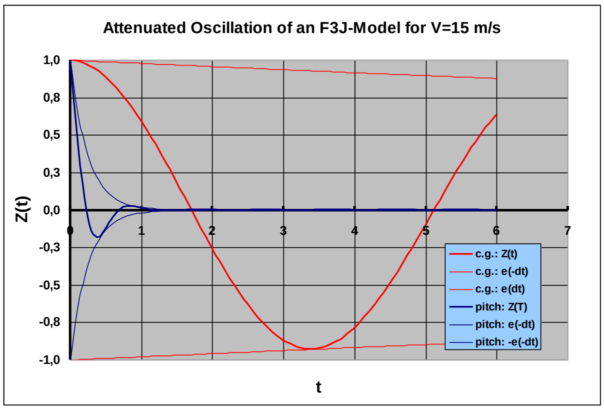
$\omega_{o,c.g.} \approx 1.80$ s^{-1},

$\delta_{c.g.} \approx 0.030$ s^{-1}

$\omega_{c.g.} \approx 1.80$ s^{-1}

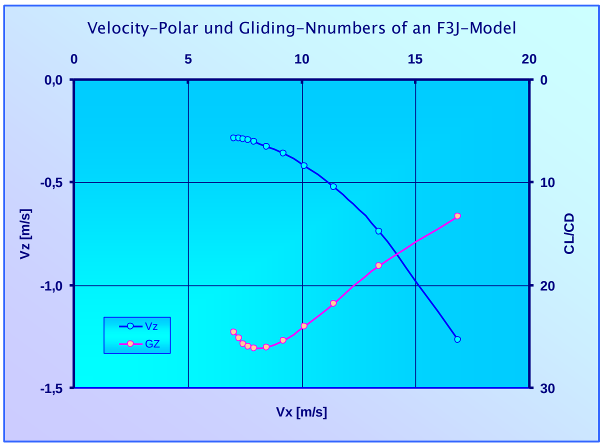
The graphic above shows that already after one period the fast pitching-oscillations come to rest and the glider finds back to the stationary flight state. At low flight-velocity the slow c.g.-oscillations of the F3J-Model are only moderately damped. However, in flight practice these long c.g.-oscillations are usually not a problem; most pilots correct them intuitively with the elevator-control of the RC-transmitter.

With increasing flight-velocity also the gliding-angle increases. As can easily be taken from the above formulae, the



dampening constant $\delta_{c.g}$ decreases slightly with increasing V and ϑ, while the oscillation frequency decreases.

For example, in the left-hand side graphic the pitch and c.g.-oscillations are given for V= 15 m/s and ϑ ≈ 3.8°.

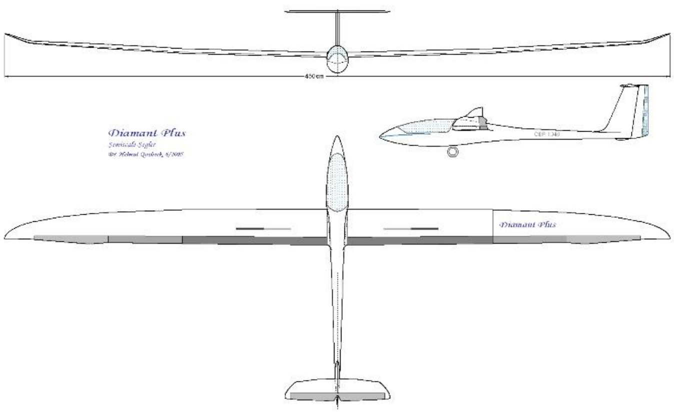


Finally, the left graphic provides the quasi-stationary theoretical performance parameters of the complete F3J-model for its expected operational flight range under inclusion of all sorts of drag related to the model.

7.4 Example of a functional Soaring Model, "DIAMANT PLUS"

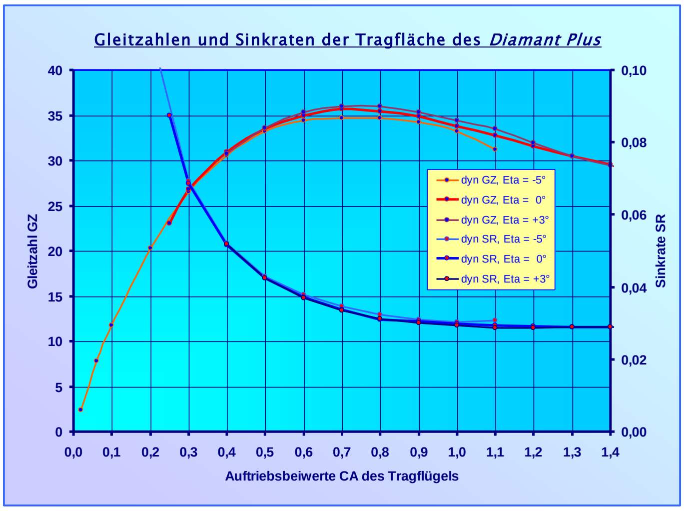
Below is given the 3-side view of the *"Diamant Plus"* soaring-model, a functional glider model of the author for thermal and alpine slope soaring and model trekking, designed and built in 2005/06. The model had been equipped with an electric motor, used for launching and/or as emergency return aid on the ground and in the mountains.

This model is a further development of a similar model which was first launched in the early eighties.



From the original *"Diamant"* the fuselage was used for practical reasons und thus the momentum-arm of the model was predetermined. In our home-page www.hq-modellflug.de one can find all details about the design aspects for the new development.

The airfoil chosen for the lifting wing is the *HQ/W-3.5/13* from the wing root till the ends of the ailerons. From there towards the tips of the wing the sections were lofted to the *HQ/Winglet*-airfoil and twisted by about -0.7° in order to achieve good-natured stall behaviour. As can be seen in the graphic, the model is equipped with flaps and flaperons which



allow to deflect the wing-sections as desired for any flight state from very slow to very high speed. By means of the left graphic showing gliding numbers and sinkrates for the lifting wing of the *"Diamant Plus"* including induced drag, the aerodynamic working point was chosen to be at c_l = 1.2 for good thermal soaring.

The geometric and aerodynamic characteristics (calculated by the ***FMFM***-program and the Eppler *PROGRAM06*) of the model relevant for the calculation of the longitudinal flight-stability are subsequently summarized:

Total mass of the model	$m \approx$	8 kg
Mean chord of the wing	$\hat{c}$ =	203.6 mm
Lifting area of the wing	A =	0.9162 m^2
Load /unit-area	$m/A \approx$ 8.8 kg/m^2	
Aspect-ratio of the wing	Λ_w =	22.1
Lift-efficiency of the wing	**a_w =**	**0.924**
Momentum-coefficient (**c_l=1.2**)	c_{Mo} =	−0.135 (including a share of the fuselage)

The centre of gravity for the chosen optimum working-point results to be at

Centre of gravity $X_{c.g.} = X_{Nw} - c_{Mo}/(a_w \cdot c_l) \cdot \hat{c}$

$= 0.0682$ mm + 0.122 · 0.2036 mm = 93.0 mm

Here from the minimum possible flight-velocity turns out to become

Velocity for the working point $V \approx 4 \cdot ((m/A)/(a_w \cdot c_l(1.2))^{1/2} = 11.3$ m/s

Based on the fuselage-dimensions of the old ***"Diamant"*** a close estimate for the length of the momentum-arm between the c.g. and the approximate position of the aerodynamic centre of the elevator then is

Length of momentum-arm $r_h \approx 1080$ mm

Using a weight-pendant for the expected elevator-mass in the position of the elevator, by means of the pendulum-method the mass-moment of inertia for the fuselage-elevator-combination related to the expected c.g. was experimentally determined to be J_{yf} = 1.38 $kg \cdot m^2$. The mass-centre of the wing turned out to be very close to the middle of the mean chord, and related to the c.g. it was calculated to be about J_{yw} = 4.066 · 0.026 = 0.106 $kg \cdot m^2$. Thus, the total mass-moment of inertia was expected to become

Mass moment of inertia: $J_y \approx 1.49$ kg · m^2

From the formula for the ratio of the q-derivative and the mass-moment of inertia

$$\frac{c_{m,\omega y}}{J_y} = -\frac{1}{J_y} \cdot 2 \cdot \pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h^2}{\hat{c}^2}$$

it can be taken that size A_h and shape a_h of the elevator are the only parameters left for adjustment of the necessary dynamic longitudinal stability-measure $c_{m,\omega y}/J_y$, since in particular the contribution of the elevator to the mass-moment of inertia growth with the length of the momentum-arm according to $J_{yh} \sim m_h \cdot r_h^2$. Because of the generally higher flight velocity of heavier models different from low-weight models like such for F3J-purposes a ratio of $c_{m,\omega y}/J_y \approx -10$ will provide sufficient attenuation of fast pitching-oscillations after disturbances as will be shown later on. In order to achieve good lift-efficiency, a double tapered elevator-shape was chosen and finally the elevator-characteristics became

Mean elevator-chord	$\hat{c}_h$ =	124.6 mm
Lifting-area of elevator	A_h =	0.0885 m^2
Aspect ratio of elevator	Λ_h =	5.7
Lift efficiency of elevator	a_h =	0.76
Pitching-attenuation	$c_{m,\omega y}/J_y$ =	8.7

In order to find out which static stability will result from the chosen elevator-characteristics, next the position of the overall aerodynamic centre of the glider is to be determined by means of the formula

$$\frac{\Delta X_N}{\hat{c}} = \frac{a_w \cdot a_h \cdot A_h / A}{1 + a_w \cdot a_h \cdot A_h / A} \cdot \frac{r_h}{\hat{c}}$$

By means of the *FMFM*-program the position of the aerodynamic centre of the lifting wing was found to be at $X_{Nw}/\hat{c} = 0.335$, and with the characteristic values given before we get

Aerodynamic centre of glider $X_N/\hat{c} \approx$ 0.335 + 0.333 = 0.6675

The static longitudinal stability-measure for the chosen c.g. corresponding to the working point $c_l = 1.2$ turns out to be

Static stability: $\boldsymbol{\sigma} = (\Delta X_N - \Delta X_S)/\hat{c} = 0.333 - 0.122 = 0.211$!

This is an appropriate measure for the longitudinal stability of a dynamic larger model.

Finally, the oscillatory behaviour of the glider after disturbances is of interest.

For the fast-pitching oscillations an appropriate modification of equations 6.1.35 and 6.1.37 provides

$$\omega_o \approx \sqrt{-\frac{1}{J_y} \cdot \left(2\pi \cdot a_w \frac{X_{c.g.} - X_{Nw}}{\hat{c}} - (1 - \frac{4 \cdot a_w}{\Lambda}) \cdot 2\pi \cdot a_h \cdot \frac{A_h}{A} \cdot \frac{r_h}{\hat{c}} \right) \cdot \frac{\rho}{2} \cdot A \cdot \hat{c} \cdot V}$$

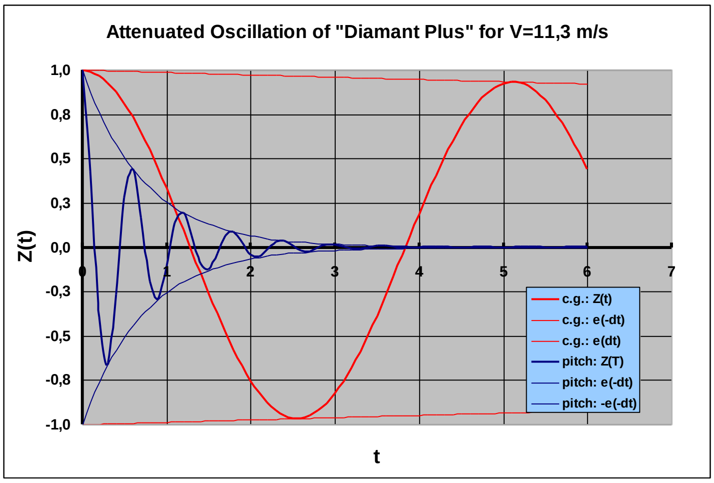
$$\delta = \frac{\pi}{2 J_y} \cdot a_h \cdot A_h \cdot r_h^2 \cdot (1 + \frac{4 \cdot a_w}{\Lambda}) \cdot \rho \cdot V$$

Taking into account the foregoing characteristic values of the model we receive

"Eigen"-frequency of pitch-oscillations	$\boldsymbol{\omega}_o = 0.944 \cdot V\ [s^{-1}]$
Damping-constant of pitch-oscillations	$\boldsymbol{\delta} = 0.121 \cdot V\ [s^{-1}]$
Frequency of pitch-oscillations	$\boldsymbol{\omega} = (\boldsymbol{\omega}_o^2 - \boldsymbol{\delta}^2)^{1/2} = 0.936 \cdot V\ [s^{-1}]$

For the minimum velocity V ≈ 11.3 m/s at the chosen optimum working conditions of the *"Diamant Plus"*, namely c_l = 1.2 for fast pitching-oscillations results: $\omega_o = 10.67\ s^{-1}$, $\delta = 1.36\ s^{-1}$, $\omega = 10.58\ s^{-1}$

Accordingly with $\vartheta \approx 1.79°$ for the slow c.g.-oscillations at the optimum working point c_l = 1.2 we have $\omega_{o,c.g.} = 1.227\ s^{-1}$, $\delta_{c.g.} = 0.0136\ s^{-1}$, $\omega_{c.g.} = 1.227\ s^{-1}$.

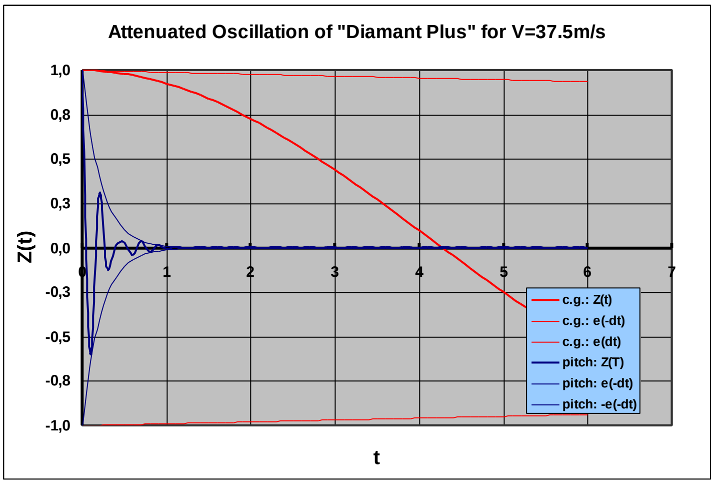


Here we have the typical behaviour of a model with a rather high mass-moment of inertia. While for the previous lower-weight F3J-model the fast pitching-oscillations already came to rest after about one cycle, here it takes about 4 - 5 cycles.

As to be expected, the attenuation of the slow c.g.-oscillations is of the same order of magnitude.

Although the attenuation of the pitch oscillation appears to be lower boarder, extensive flight practice with the *"Diamant Plus"* over a full season on flat ground and in the mountains have proven that this in no way is insufficient.

Without further explanations in the last graphic the fast pitch and slow c.g.-oscillations and their attenuation after disturbances are given as they will appear at higher flight-velocity:



As we see, the fast pitching-oscillations very soon come to rest, whilst the c.g.-oscillations take a long time and can easily be balanced out by RC-control.

The ***major conclusions*** *which can be drawn from this example for glider-models with higher mass-load are*

- *The mass-moment of inertia should be kept as low as possible in order to achieve the best possible dynamic longitudinal stability, in particular this holds true for acrobatic-gliders,*
- *As pointed out repeatedly, the weight of the model tail should be kept as low as possible, because the tail has the largest distance of all parts to the c.g. and thus contributes most to the mass-moment of inertia,*
- *For scale-gliders the size and the shape of the elevators are given by the original. It often happens, that these elevator-proportions are not sufficient for a stable flight-behaviour, as they do - in particular at slow flight - not provide the required contribution for scale-models. In such cases it may not disturb the scale impression when the model-elevator is increased by 10 to 15 %.*

- *At functional models with higher load the dynamic stability can be influenced by the length of the momentum-arm as well as by shape and size of the elevator. While designing such a model it has always to be kept in mind that attenuation by the elevator is counterbalanced by its mass moment of inertia!*

7. Final Recommendations

Whilst the longitudinal stability behaviour of the above ***F3J***- and "***Diamant Plus***"-examples was determined, it was already indicated how this could best be performed. Concluding, recommendations will be given for a more universal proceeding for the design of a plane with required longitudinal stability behaviour.

➢ Design of a functional plane

1. When designing a new functional model as for F3-classes, acrobatic flying, or free just-for-fun-flying, the first step should be to determine the dimensions and shape for the lifting wing. Thereby usually major attention should be paid to a good lift-efficiency of the wing, expressed by the shape factor a_W. E.g. this efficiency can exactly be calculated by means of the ***FMFM***-program of the author, for a quasi-elliptical wing-shape it is approximately given by consideration of the aspect ratio: $a_W \approx \Lambda_W/(2+(\Lambda_W^2+4))^{1/2}$.

2. In a second step the quasi-stationary c_l-c_d-polar corresponding to the expected wing-load m/A of the plane should be determined for the chosen airfoils of the lifting wing. From these polars the corresponding quasi-stationary sink rates and c_L/c_D-ratios for the lifting wing can be developed as functions of c_l, where c_D should include the airfoil- and the induced drag of the wing. As shown for the examples above, from these curves the optimum c_l-working-point can be determined either for best gliding-angle, minimum sinkrate or some c_l-location in between.

3. In a third step the position for the centre of gravity c.g. should be fixed according to equation 7.3.6 in section 7.3.c. As discussed earlier, at ***X-FOIL***-analyses of the wing-sections the calculated position of X_{Nw} often does not well coincide with practical experience, while the Eppler-***PROFIE06***-program supplies reliable results which are close to the quarter-point of the MAC. For normal airplanes, with sufficient accuracy the c.g. can be chosen according to $X_{c.g.}/\hat{c} = 0.25 - c_{Mo}/c_{Lw}$ for the optimum lift coefficient . This c.g. choice also leaves room for flight states with non-zero lift at the elevator and in particular for the up and down deflection of flaps.

4. In a fourth step, next the value for the static stability measure $\sigma = (X_N - X_{c.g.})/\hat{c}$ needs to be chosen. This measure is often also given in percentages of the MAC. According to experience lower weight models will already fly quite stable with 10 % stability, however, models with higher weight should better have 15 - 20 % static stability or even more. With chosen σ and $X_{c.g.}$ the necessary position of the overall aerodynamic centre X_N for the required static stability follows. Then, by means of equation 4.17, and $r_{Nh} \approx r_h$ an idea for the size A_h of the elevator and its momentum arm r_h can be developed as shown in the examples. For aerodynamic reasons, namely in order to keep the drag of the elevator as low as possible, it may be advisable to choose the elevator area A_h as small as the aerodynamic characteristics of the elevator-airfoil allow and to compensate this with a longer momentum arm. E.g. for larger F3J-models $A_h/A \approx 0.09$ would be sufficient in order to achieve appropriate aerodynamic elevator performance with the airfoil *HQ/W-0/9*.

5. In the fifths step at least a rough idea should be developed for the mass-moments of inertia J_y related to the c.g., in particular also for that of the tail part which contributes most to the overall value according to $J_{y;tail} \sim m_{tail} \cdot r_{tail}^2$. It cannot be repeated often enough, the weight of the tail and the rear part of the fuselage should be as low as possible in order to achieve good dynamic longitudinal stability.

6. Once the c.g. and the overall aerodynamic centre X_N are defined by the choice of the elevator dimensions $a_h \cdot A_h$ and its distance r_h from the c.g., then with the estimated mass-moment of inertia J_y the attenuation of disturbances of the angle of attack, gliding angle and velocity are also determined. A closer

look to the formulas for the fast pitch-oscillations appearing after disturbance of the angle of attack tells us that both the pitching-moment $c_{m,\omega y} \sim A_h \cdot r_h^2$ and the major contribution to the mass-moment of inertia $J_{yh} \approx m_h \cdot r_h^2$ depend on the second power of r_h . Consequently, the major contributions to the attenuation coefficient of the fast pitching-oscillations result from size and mass of the elevator and changes proportionally to the flying velocity according to

$$\delta \sim A_h / m_h \cdot V$$

This again demonstrates how important the weight of the elevator (and that of fin and rear fuselage as well) is for fast damping of pitch-disturbances. (*Here from the author's preference for light-weight V-tails originates.*) Accordingly, the frequency of the fast pitching-oscillations is mainly determined by

$$\sqrt{A_h / m_h} \cdot \sqrt{1/r_h} \cdot V$$

The frequency of the fast-pitching oscillations increases with the square-root of the elevator size and with the velocity while it decreases with the square-roots of the tail-weight and the elevator-momentum-arm. Thus, as already required for other reasons before, a smaller elevator and a longer momentum arm as required for other reasons before will help to keep the oscillation-frequency low. Although a higher tail-weight would reduce the oscillation-frequency, for reasons mentioned before, low tail weight is to be preferred.

➢ Design of a Scale-Model

1. When designing a scale-model, the first step should be to determine the dimensions and shape for the lifting wing and the elevator from the corresponding data of the original. Therefrom the lift-efficiency-factors a_w and a_h are to be determined. E.g. this efficiency can exactly be calculated by means of the ***FMFM***-program of the author, for a quasi-elliptical wing-shape (which is applied for almost all modern gliders) they can approximately be calculated by means of the aspect ratios:

$$a_w \approx \Lambda_w/(2+ (\Lambda_w^2 + 4))^{1/2} \text{ and } a_h \approx \Lambda_h/(2+ (\Lambda_h^2 + 4))^{1/2}.$$

2. In a second step like for the functional planes the quasi-stationary c_l-c_d-polar corresponding to the expected wing-load m/A should be determined for the airfoil of the lifting wing. From these polars the corresponding quasi-stationary sink rates and c_L/c_D-ratios for the lifting wing can be developed as functions of c_l, where c_D should include the airfoil- and the induced drag of the wing. As shown for the examples above, from these curves the optimum c_l-working-point can be determined either for best gliding-angle or minimum sinkrate of the scale-plane.

3. In a third step the position for the centre of gravity $X_{c.g.}$ should be determined according to equation 7.3.6 in section 7.3.c. In general for normal planes with sufficient accuracy the c.g. can be chosen according to $X_{c.g.}/\hat{c} = 0.25 - c_{Mo}/c_{Lw}(opt)$. Again, as before this c.g. choice also leaves room for flight states with non-zero lift at the elevator and in particular for the up and down deflection of flaps.

4. As soon as the position of the c.g. is determined, the length of the momentum-arm r_h (the distance of the aerodynamic centre of the elevator from the c.g.) can be determined, and based on the geometric data of the model and the wing and elevator efficiencies, a_w and a_h, the position of the overall aerodynamic centre of the scale-model can be found by means of equation 4.1.17, and finally the static stability measure σ. If the static stability of a larger scale-glider should turn out to be $\sigma < 0.18$ then the stall behaviour of the model at slow soaring may become critical. In such cases an enlargement of the elevator-size should be considered, since this would not harm the scale impression and would be the easiest way to improve the static stability. Unfortunately, the static stability of Old-timer-glider often also suffers from too short distances of the elevator from the c.g., in these cases it may be advisable to also lengthen a bit the rear fuselage-part.

5. In the next step at least a rough idea should be developed for the mass-moments of inertia J_y related to the c.g., in particular also for that of the tail parts which contribute most to the overall value according to $J_{y;tail} \sim m_{tail} \cdot r_h^2$. The weight of the tail and the rear part of the fuselage should be as low as possible in order to achieve good dynamic longitudinal stability. In particular larger scale models often

carry a lot of unnecessary weight along in their tail parts. E.g., since large scale-gliders offer much space in the fin, unfortunately often heavy, strong servos are mounted therein.

6. Once the c.g. is defined, then with the estimated mass moment of inertia J_y the attenuation of disturbances of the angle of attack, gliding angle and velocity can be derived and it can be seen which dynamic stability behaviour the scale model may develop for certain flight-conditions. However, there is no further parameter to be found by these considerations which could be changed to influence the dynamic flight behaviour.